ISSN:2153-182X, E-ISSN: 2153-1838
Vol. 17 No. 02 (2023)

3 -CONSTRAINED TOTAL LABELING OF A GRAPH

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Abstract

A 3 - Constrained total labeling of a graph $G(V, E)$ is a bijective mapping $g : V \cup E \rightarrow \{1, 2, 3, . . . , |V| + |E|\}$ with extra constraints $|g(u) - g(v)| \geq 3$, $|g(u) - g(uv)| \geq 3$ and $|g(uv) - g(vw)| \geq 3$ whenever $u, v, w \in V$ and $uv, vw \in E$. *A* graph *G* which admits such labeling is called a 3-Constrained total graph. In this paper we determine that $C_n \times P_2$, Double triangular snake graph, Chain sum graph of first kind, Helm graph, Sunlet graph, Wheel graph, Gear graph, Ladder graph, Dutch windmill graph, Triple triangular snake graph, Zig-Zag Graph and Double squared chain graph are 3 -Constrained total graphs.

Keywords: Graph Labeling, Total Labeling, 3-Constrained Total Graph. 2000 Mathematics Subject Classification: 05C78

1 Introduction

Graph theory is the rapid expanding area of combinatorics where labeled graph constitute a very convenient mathematical version for the wide collection of applications. A labeling of a graph is a function from set of vertices or edges or both to set of integers, subject to certain constraints. Kotzig and Rosa [1] introduced and proved that C_p for all $p > 3$, $K_{l,m}$ for all p , and p has an edge magic total labeling. Enomoto et al. [2] defined and established that $K_{1,p-1}$, if p is odd then C_p graph and caterpillars are super edge magic. Furthermore, it was shown that $q \leq 2p - 3$, if a graph has p vertices with q edges represents super edge magic. Also proved that if a graph with p vertices and q edges is super edge magic then $q \leq 2p - 3$.

MacDougall et al. in 1999 [3] and established that P_n, C_p for $(p > 2)$, $K_{l,\,l}$ for $(l > 1)$ and K_p (p is odd) have vertex magic total labelings and also proved that if $p > q + 1$, then $K_{l,m}$ does not satisfy vertex magic total labeling. Exoo et al. [4] considered a total magic labeling of a graph $G(V, E)$ which is both an edge magic and a vertex magic total labeling.

Baca et al. [5] established the concept of (a, d) -vertex antimagic total labeling for a graph $G (V,E)$. They proved that paths, prisms & generalized Petersen graphs have (a, d)-vertex antimagic total labelings. An (a, d) -edge antimagic total labeling for a graph G(V, E) was described by Simanjuntak et al. in [6]. In order to label paths and cycles for various values of a and d, they established" (a, d) -edge antimagic total labeling."

Recently Shreedhara K. et al. [7] established Smarandechely k-constrained total labeling. Motivated by above labeling, we consider 3-constrained total labeling of some

ISSN:2153-182X, E-ISSN: 2153-1838
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family of graph G and establish the same for $C_p \times P_2$, Helm graph, Double triangular snake graph, Sun graph, Chain sum graph of first kind, Wheel graph, Gear graph, Ladder graph, Dutch windmill graph, Triple triangular snake graph Zig-Zag Graph and Double squared chain graph.

2 Definitions of Some Classic Graphs:

Definition 2.1. The triangular snake T_p is attained by substituting a cycle $C3$ at each edge of a path P_p.

Definition 2.2. The alternate triangular snake $A(T_p)$ is attained from a path P_p by substituting each alternate edge of P_p by a cycle C_3.

Definition 2.3. Two alternate triangular snakes with a shared path make up the alternate double triangular snake $DA\ (T_p)$.

Definition 2.4. Two alternate triangular snakes with a shared path make up the alternate double triangular snake $DA\ (T_p)$.

Definition 2.5. The quadrilateral snake Q_p is collected from a path P_p by substituting each edge of P_p by a cycle C_4.

Definition 2.6. The double quadrilateral snake is denoted by DQ_p is created by two quadrilateral snakes that having a shared path.

A 3 - Constrained total labeling of a graph $G\ (V, E)$ is a bijective mapping $g : VUE \rightarrow \{1, 2, \ldots, |V| + |E|\}$ with the extra requirements that

$$| g(u) - g(v) | \geq 3$$
$$| g(u) - g(uv) | \geq 3$$
$$| g(uv) - g(vw) | \geq 3$$

whenever $u, v, w \in V$ and $uv, vw \in E$. Graph G is referred to as a
3 -Constrained total graph if such labeling is allowed.

3 Main Outcomes of Cycle Related Graphs

Theorem 3.1. For $p \geq 5$, $C_p \times P_2$ *is a 3 -constrained total graph.*

Proof. $C_P \times P_2$ is a graph with $V = \{u_j , v_j : 1 \leq j \leq p\}$ and $E = \{v_jv_j+1, u_ju_{j+1}: 1 \leq j \leq p - 1\} \cup \{v_nv_1, u_nu_1\} \cup \{u_jv_j : 1 \leq j \leq p\}$ where $|V| = 2p$ and $|E| = 3p$. Indicate the total labeling $g : V\,UE \rightarrow \{1, 2, \ldots, 5p\}$ on a graph $C_p \times P_2$ as

$$\left.\begin{aligned} g(u_j) &= 5j - 1 \\ g(v_j) &= 5j - 4 \end{aligned}\right\} 1 \leq j \leq p$$

$$\left.\begin{aligned} g(v_{j+1}v_{j+2}) &= 5j - 3 \\ g(u_{j+1}u_{j+2}) &= 5j - 2 \end{aligned}\right\} 1 \leq j \leq (p-2)$$

$$g(u_{j+4}v_{j+4}) = 5j for 1 \leq j \leq p - 4$$

$$g(u_jv_j) = 5p + 5j - 20 \ \ for 1 \leq j \leq 4$$

$g(v_pv_1) = 5p - 8$, $g(v_1v_2) = 5p - 3$, $g(u_pu_1) = 5p - 7$ $g(u_1u_2) = 5p - 2$.

Now it is easy to verify that $C_p \times P_2$ satisfy 3 – Constrained total labeling.

Theorem 3.2. *Helm* H_p*, for* $p \geq 3$ *is a 3 - constrained total Graph.*

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Proof. Helm graph consists of $V = \{v_0, v_j, u_j : 1 \leq j \leq p\}$ and $E = \{v_0v_j, u_jv_j : 1 \leq j \leq p\} \cup \{v_jv_j+1 : 1 \leq j \leq p-1\} \cup \{v_1v_p\}$ with $|V| = 2p+1$ and $|E| = 3p$. Consider a total labeling $g : V \cup E \rightarrow \{1, 2, . . . , 5p + 1\}$ defined on Hp by

$$g(v_0\, v_j) = 3j - 2 \text{ for } 1 \leq j \leq p$$

$$\left.\begin{aligned} &g(v_{j+4}\, v_{j+5}) = 3j \\ &g(v_{j+5}\, u_{j+5}) = 3p + j \\ &g(u_{j+5}) \end{aligned}\right\} 1 \leq j \leq p-5$$

$$\left.\begin{aligned} g(ujvj) &= 4p + j - 5 \\ g(u_j) &= 5p + j - 5 \end{aligned}\right\} 1 \leq j \leq 5$$

$$g(v_jv_{j+1}) = 3p + 3j - 12 \text{ for } 1 \leq j \leq 4$$

$g(v_1) = 3n - 4$, $g(v_2) = 3n - 1$,
$g(v_0) = 5n + 1$, $g(v_pv_1) = 3p - 12$

Here the function g serves as a 3 - Constrained total labeling for H_p.

Theorem 3.3. *Sunlet graph is a 3 -constrained total Graph.*

Proof. Let G (V, E) be n-sunlet graph hold $V= \{v_j, u_j : 1 \leq j \leq p\}$ *and* $E= \{v_jv_{j+1} : 1 \leq j \leq p - 1\} \cup \{v_ju_j : 1 \leq j \leq p\} \cup \{v_pv_1\}$ *with* $|V| = 2p$ *and* $|E| = 2p$ edges. Consider a total labeling $g : V \cup E \rightarrow \{1, 2, . . . , 4p\}$ defined on vertices by

$$\left.\begin{aligned} g(v_j) &= 3p + 3j - 13 \\ g(u_j) &= 4p + j - 4 \end{aligned}\right\} 1 \leq j \leq 4$$

$$\left.\begin{aligned} g(v_{j+4}) &= 3j - 1 \\ g(u_{j+4}) &= 3n + j \end{aligned}\right\} 1 \leq j \leq p-4$$

Now defined on edges by

$$g(v_jv_{j+1}) = 3j - 2 \text{ for } 1 \leq j \leq p - 1$$
$$g(v_j{+}3u_j + 3) = 3j \text{ for } 1 \leq j \leq p - 3$$
$$g(u_jv_j) = 3p + 3j - 9 \text{ for } 1 \leq j \leq 3$$
$$g(v_pv_1) = 3p - 2$$

Here the Sunlet graph admits 3 - Constrained total labeling.

Theorem 3.4. *Wheel* W_p *, for* $n \geq 6$ *is a 3 -constrained total graph.*

Proof. The wheel graph is formed of $V=\{v_j : 0 \leq j \leq p\}$ *and* $E = \{v_0v_j : 1 \leq j \leq p\} \cup \{v_jv_{j+1} : 1 \leq j \leq p - 1\} \cup \{v_pv_1\}$ with $|E| = p + 1$ and $|E| = 2p$. Describe a total labeling $g : V \cup E \rightarrow \{1, 2, . . . , 3p + 1\}$ on vertices of W_p by

$$\left.\begin{aligned} g(v_0) &= 3p + 1 \\ g(v_1) &= 3p - 4 \\ g(v_2) &= 3p - 9 \\ g(v_{j+2}) &= 3j - 1 \end{aligned}\right\} 1 \leq j \leq p-2$$

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Define on edges by

$$g(v_0v_j) = 3j - 2 \text{ for } 1 \leq j \leq p$$
$$g(v_{j+4}v_{j+5}) = 3j \text{ for } 1 \leq j \leq p - 5$$
$$g(v_{j+1}\, v_{j+2}) = 3p + 3j - 9 \text{ for } 1 \leq j \leq 3$$
$$g(v_nv_1) = 3p - 12,\ g(v_1v_2) = 3p - 1$$

Here W_p admit 3 – Constrained total labelng.

Theorem 3.5. *Gear graph is a 3 -constrained total labeled graph.*

Proof. The Gear graph has $V = \{v_j : 1 \leq j \leq 2p + 1\}$ and = $\{v_1v_{2j},\ v_{2j}v_{2j}+1,\ v_{2j+1}v_{2j+2} : 1 \leq j \leq p\}$ where $|V| = 2p + 1$ and $|E| = 3p$. Consider a total labeling $g : V \cup E \to \{1, 2, \ldots, 5p + 1\}$ *into two cases.*

Case 1: If p ≡ 0 (mod 2) then,

$$\left.\begin{aligned} g(v_1) &= 5n + 1 \\ g(v_{2j}) &= j \end{aligned}\right\} 1 \leq j \leq p$$

$$\left.\begin{aligned} g(v_{4j-1}) &= p + 2j - 1 \\ g(v_{4j+1}) &= p + 2j \end{aligned}\right\} 1 \leq j \leq \frac{p}{2}$$

We label the edges

$$\left.\begin{aligned} g(v_{4j-2}\, v_{4j-1}) &= 3j + \frac{7p+4}{2} \\ g(v_{4j}\, v_{4j+1}) &= 3j + \frac{7p+6}{2} \\ g(v_{4j+3}\, v_{4j+4}) &= 3j + 2p \\ g(v_{4j+1}\, v_{4j+2}) &= 3j + 2p - 1 \end{aligned}\right\} 1 \leq j \leq \frac{p}{2} - 1$$

$$g(v_1v_{2j}) = 2p + 3j - 2 \text{ for } 1 \leq j \leq p$$

$$g(v_2v_{2p+1}) = \frac{7p-2}{2},\ g(v_3v_4) = \frac{7p}{2},\ g(v_{2p-2}\, v_{2p-1}) = \frac{7p+4}{2},$$
$$g(v_{2p}v_{2p+1}) = \frac{7p+6}{2}$$

Now it is easy to verify that G_p satisfy 3 -Constrained total labeling.

Case 2: If p ≡ 1 (mod 2), then

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$$\left.\begin{aligned} g(v_1) &= 5n+1 \\ g(v_{2j}) &= j \end{aligned}\right\} 1 \le j \le p$$

$$g(v_{4j-1}) = p + 2j - 1 \ for\ 1 \le j \le \frac{p+1}{2}$$

$$g(v_{4j+1}) = p + 2j \ \ for\ 1 \le j \le \frac{p-1}{2}$$

We label the edges

$$g(v_1v_{2j}) = 2p + 3j - 2 \ for\ 1 \le j \le p$$

$$\left.\begin{aligned} & g(v_{4j+1}\ v_{4j+1} = 3j + 2p - 1) \\ & g(v_{4j-2}\ v_{4j-1}) = 3j + \frac{7p+1}{2} \\ & g(v_{4j}\ v_{4j+1}) = 3j + \frac{7p+3}{2} \end{aligned}\right\} 1 \le j \le \frac{p-1}{2}$$

$$g(v_{4j+3}\ v_{4j+4}) = 3j + 2p \ for\ 1 \le j \le \frac{p-3}{2}$$

$$g(v_{2p}v_{2p+1}) = \frac{7p+3}{2},\ g(v_3v_4) = \frac{7p+1}{2},\ g(v_2v_{2p+1}) = \frac{7p-3}{2}.$$

This labeling establish a 3 -Constrained total labeling for G_p
Illustrative Example for G_8 graph

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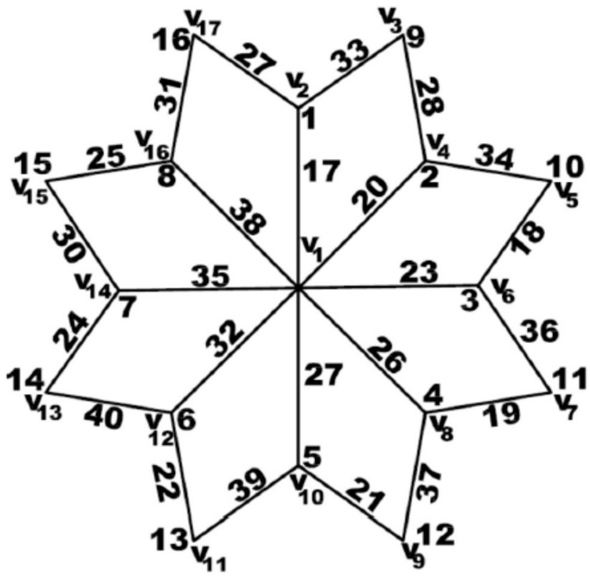

Figure 1: 3 constrained total labeling of Gear graph G_n

Theorem 3.6. *Ladder graph L_p is a 3 -constrained total labeled graph.*

Proof. The Ladder graph comprises of set $V=\{v_j : 1 \le j \le 2p\}$ and set $E = \{v_{2j-1}v_{2j} : 1 \le j \le p\} \cup \{v_{2j-1}v_{2j+1},\ v_{2j}v_{2j+2} : 1 \le j \le p - 1\}$ with $|V| = 2p$ and $|E| = 3p - 2$. Hence we consider a total labeling $g : V \cup E \to \{1,2, . . . , 5p - 2\}$ on Lp in to two events.

Case 1: If p = 2k then,

$$\left.\begin{aligned} g(v_{4j-3}) &= p + 2j - 1 \\ g(v_{4j-2}) &= p + 2j + 3 \\ g(v_{4j-1}) &= 2j + 2p \\ g(v_{4j}) &= 2j + p \end{aligned}\right\} 1 \le j \le \frac{p}{2}$$

We label the edges

$$g(v_{2j-1}v_{2j}) = j \text{ for } 1 \le j \le p$$

$$\left.\begin{aligned} g(v_{4j-3}v_{4j-1}) &= 3p + 2j - 1 \\ g(v_{4j-2}v_{4j}) &= 4p + 2j - 2 \end{aligned}\right\} 1 \le j \le \frac{p}{2}$$

$$\left.\begin{aligned} g(v_{4j-1}v_{4j+1}) &= 4p + 2j - 1 \\ g(v_{4j}v_{4j+2}) &= 3p + 2 \end{aligned}\right\} 1 \le j \le \frac{p}{2} - 1$$

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This labeling serves as a 3 -constrained total labeling for L_p.
Case 2: If $p = 2k + 1$ then,

$$\left.\begin{aligned} g(v_{4j-3}) &= p + 2j - 1 \\ g(v_{4j-2}) &= 2p + 2j - 1 \end{aligned}\right\} 1 \le j \le \frac{p+1}{2}$$

$$\left.\begin{aligned} g(v_{4j-1}) &= 2p + 2j \\ g(v_{4j}) &= 2j + p \end{aligned}\right\} 1 \le j \le \frac{(p-1)}{2}$$

Label the edges

$$\left.\begin{aligned} g(v_{4j-3}v_{4j-1}) &= 2j + 3p - 1 \\ g(v_{4j-1}v_{4j+1}) &= 2j + 4p - 1 \\ g(v_{4j-2}v_{4j}) &= 2j + 4p - 2 \\ g(v_{4j}v_{4j+2}) &= 2j + 3p \end{aligned}\right\} 1 \le j \le \frac{p-1}{2}$$

$$g(v_{2j-1}v_{2j}) = j \text{ for } 1 \le j \le p$$

This proves that L_p is a 3 -constrained total labeled graph.

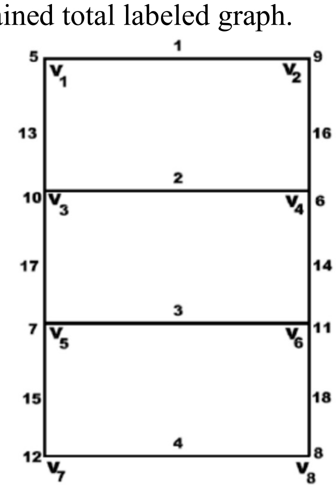

Figure 2: 3 constrained total labeling of Ladder graph L_n

Theorem 3.7. *Dutch wind mill graph* D_4^p *for* $p \ge 4$ *is a 3 -constrained total labeled* graph

Proof. Dutch wind mill graph has $V = \{v_j : 1 \le j \le 3p + 1\}$ and $E = \{v_1v_{3j-1}, v_1v_{3j+1}, v_{3j-1}v_{3j}, v_{3j}v_{3j+1} : 1 \le j \le p\}$ where |V |=3p+1 and |E|= 4p.

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Let us consider a total labeling $g : V \cup E \longrightarrow \{1, 2, . . . 7p+1\}$ defined on vertices by

$$\left.\begin{aligned} g(v_1) &= 7p+1 \\ g(v_{3j-1}) &= 6p+6j-15 \\ g(v_{3j+1}) &= 6p+6j-12 \\ g(v_{3j}) &= 7p+j-2 \end{aligned}\right\} j = 1, 2$$

$$\left.\begin{aligned} g(v_{3j+5}) &= 6j-3 \\ g(v_{3j+7}) &= 6j \\ g(v_{3j+6}) &= 6p+j \end{aligned}\right\} 1 \le j \le p-2$$

Defined on edges by

$$\left.\begin{aligned} g(v_1v_{3j-1}) &= 6j-5 \\ g(v_1v_{3j+1}) &= 6j-2 \\ g(v_{3j+6}) &= 6n+j \end{aligned}\right\} 1 \le j \le n$$

$$\left.\begin{aligned} g(v_{3j+2}v_{3j+3}) &= 6j-4 \\ g(v_{3j+3}v_{3j+4}) &= 6j-1 \end{aligned}\right\} 1 \le j \le p-1$$

$$g(v_{j+1}v_{j+2}) = 6p + 3j - 7 \text{ for } j = 1, 2$$

This labeling serves as a 3 -constrained total labeling for D_4^P.

4 Outcomes of Snake Related Graphs

Theorem 4.1. *Double triangular snake graph DT_p for $p \ge 2$ is a 3 –constrained total Graph.*

Proof. Double triangular snake graph is made up of $V = \{w_j : 1 \le j \le p\} \cup \{u_j, v_j : 1 \le j \le p-1\}$ and $E = \{w_jw_{j+1}, w_ju_j, w_{j+1}u_j, w_jv_j, w_{j+1}v_j : 1 \le j \le p-1\}$
with $|V| = 3p-2$ and $|E| = 5p-5$. Consider a total labeling $g : V \cup E \to \{1, 2, . . . , 8p-7\}$ on DT_p in to three events.

Case 1. If p ≡ 0 (mod 3) then,

$$\left.\begin{aligned} g(u_j) &= j \\ g(v_j) &= j-1+p \end{aligned}\right\} 1 \le j \le p-1$$

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$$\left.\begin{aligned} g(w_j) &= 6p+6j-11 \\ g(w_{\frac{n}{3}+j}) &= 6p+6j-10 \\ g(w_{\frac{2p}{3}+j}) &= 6p+6j-9 \end{aligned}\right\} 1 \le j \le \left(\frac{p}{3}\right)$$

We label the edges

$$\left.\begin{aligned} g(w_j w_{j+1}) &= 6p+6j-8 \\ g(w_{\frac{p}{3}+j} w_{\frac{p}{3}+j+1}) &= 6p+6j-7 \end{aligned}\right\} 1 \le j \le \left(\frac{p}{3}\right)$$

$$\left.\begin{aligned} g(u_j w_j) &= j+2p-2 \\ g(u_j w_{j+1}) &= j+3p-3 \\ g(v_j w_j) &= j+4p-4 \\ g(v_j w_{j+1}) &= j+5p-5 \end{aligned}\right\} 1 \le j \le p-1$$

$$g(w_{\frac{2p}{3}+j} w_{\frac{2p}{3}+j+1}) = 6p+6j-6 \;\; for \;\; 1 \le j \le \left(\frac{p}{3}-1\right)$$

This labeling serves as a 3 -constrained total labeling for $D(T_p)$ if p = 3k.

Case 2. If p = 3k + 1, then

$$\left.\begin{aligned} g(u_j) &= j \\ g(v_j) &= p+j-1 \end{aligned}\right\} 1 \le j \le p-1$$

$$g(w_j) = 6p+6j-11 \;\; for \;\; 1 \le j \le \left(\frac{p+2}{3}\right)$$

$$\left.\begin{aligned} g\left(w_{\frac{p+2}{3}+j}\right) &= 6p+6j-7 \\ g\left(w_{\frac{2p+1}{3}+j}\right) &= 6p+6j-6 \end{aligned}\right\} 1 \le j \le \left(\frac{p-1}{3}\right)$$

We label the edges

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$$\left.\begin{aligned} g(u_j w_j) &= 2p-2+j \\ g(u_j w_{j+1}) &= 3p-3+j \\ g(v_j w_j) &= 4p-4+j \\ g(v_j w_{j+1}) &= 5p-5+j \end{aligned}\right\} 1 \le j \le p-1$$

$$\left.\begin{aligned} g(w_j`w_{j+1} &= 6p+6j-8 \\ g(w_{\frac{p-1}{3}+j} w_{\frac{p-1}{3}+j+1} &= 6p+6j-10 \\ g(w_{\frac{2p-2}{3}+j} w_{\frac{2n-2}{3}+j+1} &= 6p+6j-9 \end{aligned}\right\} 1 \le j \le \frac{p-1}{3}$$

which serves as a 3 -constrained total labeling for $D(T_p)$ if p ≡ 1(mod 3)

Case 3. If $p \equiv 2($ mod 3) then we label the vertices

$$\left.\begin{aligned} g(u_j) &= j \\ g(v_j) &= p+j-1 \end{aligned}\right\} 1 \le j \le (p-1)$$

$$\left.\begin{aligned} g(w_j) &= 6p+6j-11 \\ g\left(w_{\frac{2p-1}{3}+j}\right) &= 6p+6j-9 \end{aligned}\right\} 1 \le j \le \frac{p+1}{3}$$

$$g\left(w_{\frac{p+1}{3}+j}\right) = 6p+6j-7 \;\; for\; 1 \le j \le \frac{p-2}{3}$$

$$\left.\begin{aligned} g(u_j w_j) &= 2p-2+j \\ g(u_j w_{j+1}) &= 3p-3+j \\ g(v_j w_j) &= 4p-4+j \\ g(v_j w_{j+1}) &= 5p-5+j \end{aligned}\right\} 1 \le j \le p-1$$

The edges

$$\left.\begin{aligned} g(w_j w_{j+1}) &\quad 6p+6j-8 \\ g(w_{\frac{2p-1}{3}+j} w_{\frac{2p-1}{3}+j+1}) &\quad 6p+6j-6 \end{aligned}\right\} 1 \le j \le \frac{p-2}{3}$$

$$g\left(w_{\frac{p-2}{3}+j} w_{\frac{p-2}{3}+j+1}\right) = 6p+6j-10 \;\; for\; 1 \le j \le \frac{p+1}{3}$$

This labeling serves as a 3 -constrained total labeling for $D(T_p)$ if $p \equiv 2$ (mod 3).

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Theorem 4.2. *The chain sum graph of first kind is a 3 -constrained total Graph for $n \geq 2$*
Proof. The chain sum graph of first kind consists of vertex set
$V = \{v_j : 1 \leq j \leq p\} \cup \{u_j, w_j : 1 \leq j \leq p - 1\}$ and edge set
$E = \{u_jv_j, u_jv_{j+1}, w_jv_j, w_jv_{j+1}, u_jw_j : 1 \leq j \leq p - 1\}$ with $|V| = 3p - 2$ and
$|E| = 5p - 5$. Consider a labeling $g : V \cup E \rightarrow \{1, 2, \ldots, 8p - 7\}$ defined on vertices by.

$$\left.\begin{array}{ll} f(u_j) & j \\ f(w_j) & p + j - 1 \end{array}\right\} 1 \leq j \leq p - 1$$

$$f(v_j) = 6p + j - 6 \text{ for } 1 \leq j \leq p$$

Defined on edges by

$$\left.\begin{array}{ll} f(u_jv_j) & = 2p + j - 2 \\ f(u_jv_{j+1}) & = 3p + j - 3 \\ f(w_jv_j) & = 4p + j - 4 \\ f(w_jv_{j+1}) & = 5p + j - 5 \end{array}\right\} 1 \leq j \leq (p - 1)$$

Now it is simple to prove that chain sum graph of first kind satisfy 3 – Constrained total labeling.

Theorem 4.3. Triple triangular snake graph T(Tp) is a 3 -constrained total labeled graph.

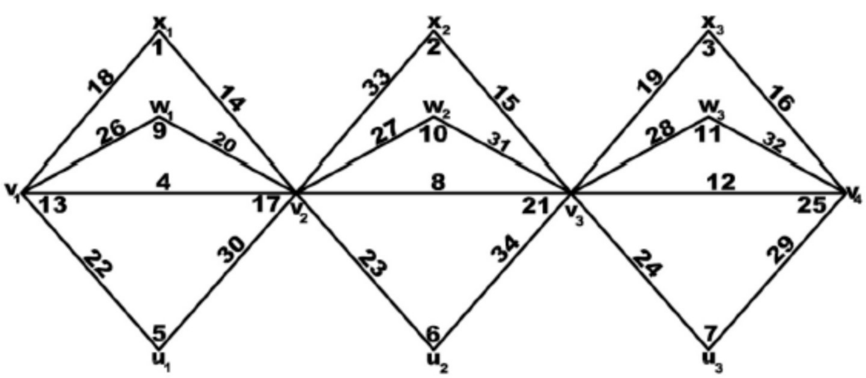

Figure 3: 3 constrained total labeling of Triple triangular snake graph $T(T_p)$

Proof. Let graph $G = T(T_p)$ be made up of vertex set $V = \{v_j : 1 \leq j \leq p + 1\} \cup \{w_j, x_j, u_j : 1 \leq j \leq p\}$ and an edge set $E = \{v_ju_j, v_{j+1}u_j, v_jw_j, v_{j+1}w_j, v_jx_j, v_{j+1}x_j, v_jv_{j+1} : 1 \leq j \leq p\}$ where, $|V| = 4p+1$ and $|E| = 7p$. Let us define a total labeling $g : VUE \rightarrow \{1, 2, \ldots 11p + 1\}$ on $T(T_p)$ into two events.

Case 1: If $p = 2k$ then,

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$$\left.\begin{aligned} g(x_{2j}) &= 3j-1 \\ g(x_{2j-1}) &= 3j-2 \\ g(u_{2j}) &= 3j-\frac{3p}{2}-1 \\ g(u_{2j-1}) &= 3j-\frac{3p}{2}-2 \\ g(w_j) &= j+5p+1 \\ g(v_{2j}) &= j+\frac{9p}{2}+1 \end{aligned}\right\} 1 \le j \le \frac{p}{2}$$

$$g(v_{2j-1}) = j+3p \ for\ 1 \le j \le \frac{p+2}{2}$$

We also define

$$\left.\begin{aligned} g(v_j x_j) &= j+7p+1 \\ g(v_{j+1} x_j) &= j+\frac{7p}{2}+1 \\ g(v_j w_j) &= j+9p+1 \end{aligned}\right\} 1 \le j \le p$$

$$\left.\begin{aligned} g(v_{j+1} w_j) &= j+10p+1 \\ g(v_j u_j) &= j+6p+1 \\ g(v_{j+1} u_j) &= j+8p+1 \\ g(v_j u_{j+1}) &= 3j \end{aligned}\right\} 1 \le j \le p$$

Here the total labeling shows that $T(T_p)$ is a 3 -constrained total labeled graph.

Case 2: If $p \equiv 1$ (mod 2) we define

$$\left.\begin{aligned} g(x_{2j}) &= 3j-1 \\ g(u_{2j}) &= 3j-\frac{3p-1}{2} \end{aligned}\right\} 1 \le j \le \frac{p-1}{2}$$

$$\left.\begin{aligned} g(x_{2j-1}) &= 3j-2 \\ g(u_{2j-1}) &= 3j-\frac{3p-5}{2} \\ g(v_{2j}) &= j+\frac{9p+1}{2} \\ g(v_{2j-1}) &= j+3p \end{aligned}\right\} 1 \le j \le \frac{p+1}{2}$$

$$g(w_j) = j+5p+1 \ for\ 1 \le j \le p$$

We also define

$$\left.\begin{aligned} g(v_jx_j) &= 7p+j+1 \\ g(v_{j+1}x_j) &= \frac{7p}{2}+j+1 \\ g(v_jw_j) &= 9p+j+1 \end{aligned}\right\} 1 \le j \le p$$

$$\left.\begin{aligned} g(v_{j+1}w_j) &= 10p+j+1 \\ g(v_ju_j) &= 6p+j+1 \\ g(v_jv_{j+1}) &= 3j \\ g(v_{j+1}u_j) &= 8p+j+1 \end{aligned}\right\} 1 \le j \le p$$

This is a 3 -constrained total labeled graph.

Theorem 4.4. Triple quadrilateral snake graph $T(Q_p)$ is a 3 -constrained total labeled graph.

Proof. Triple quadrilateral snake graph has vertex set $V = \{t_j, u_j, v_j, w_j, x_j, y_j, z_j, v_{j+1} : 1 \le j \le p\}$ and edge set $E = \{v_jv_{j+1}, v_jw_j, w_jt_j, t_jv_{j+1}, v_jz_j, z_jx_j, x_jv_{j+1}, v_ju_j, u_jy_j, y_jv_{j+1} : 1 \le j \le p\}$ with $|V| = 7p + 1$ and $|E| = 10p$. Now we consider the total labeling $g : V \cup E \rightarrow \{1, 2, ... 17p + 1\}$ into two events.

Case 1: If $p \equiv 0$ (mod 2), then

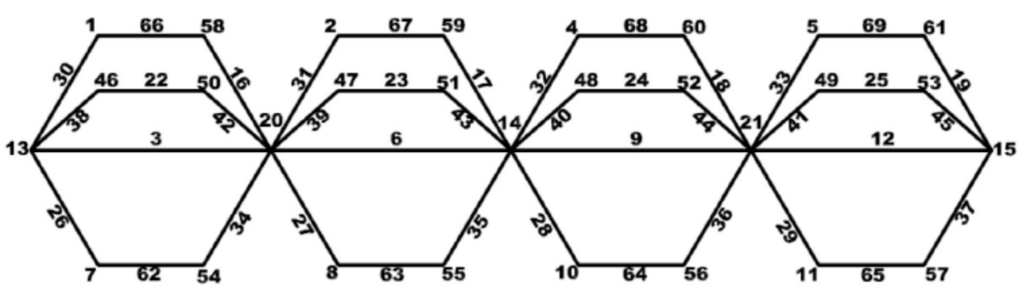

Figure 4: 3 constrained total labeling of Triple quadrilateral snake graph $T(Q_p)$

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$$\left.\begin{aligned} g(z_{2j}) &= 3j-1 \\ g(z_{2j-1}) &= 3j-2 \\ g(v_{2j}) &= j+\frac{9p}{2}+1 \\ g(u_{2j}) &= 3j+\frac{3p}{2}-1 \\ g(u_{2j-1}) &= 3j+\frac{3p}{2}-1 \end{aligned}\right\} 1 \le j \le \frac{p}{2}$$

$$g(v_{2j-1}) = j+3p \ \ for\ 1 \le j \le \frac{p}{2}+1$$

$$\left.\begin{aligned} g(x_j) &= j+14p+1 \\ g(w_j) &= j+11p+1 \\ g(t_j) &= j+12p+1 \\ g(x_j) &= j+13p+1 \end{aligned}\right\} 1 \le j \le p$$

We also define,

$$\left.\begin{aligned} g(v_j z_j) &= j+7p+1 \\ g(z_j x_j) &= j+16p+1 \\ g(x_j v_{j+1}) &= j+\frac{7p}{2}+1 \\ g(v_j w_j) &= j+9p+1 \end{aligned}\right\} 1 \le j \le p$$

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$$\left.\begin{aligned} g(w_j t_j) &= j+5p+1 \\ g(t_j u_{j+1}) &= j+10p+1 \\ g(v_j v_{j+1}) &= 3j \\ g(v_j u_j) &= j+6p+1 \\ g(u_j y_j) &= j+15p+1 \\ g(y_j v_{j+1}) &= j+8p+1 \end{aligned}\right\} 1 \le j \le p$$

Case 2: If $p \equiv 1$ (mod 2) we define

$$\left.\begin{aligned} g(z_{2j}) &= 3j-1 \\ g(u_{2j}) &= 3j+\frac{3p-1}{2} \end{aligned}\right\} 1 \le j \le \frac{p-1}{2}$$

$$g(z_{2j-1}) = 3j-2$$

$$g(v_{2j}) = j+\frac{9p+1}{2}$$

$$g(v_{2j-1}) = j+3p$$

$$g(v_{2j-1}) = 3j+\frac{3p-5}{2} \text{ for } 1 \le j \le \frac{p+1}{2}$$

$$g(x_j) = j+14p+1$$

$$g(w_j) = j+11p+1$$

$$g(t_j) = j+12p+1 \text{ for } 1 \le j \le p$$

We also define

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$$g(v_j z_j) = j + 7p + 1$$
$$g(z_j x_j) = j + 16p + 1$$
$$g(x_j u_{j+1}) = j + \frac{7p+1}{2}$$
$$g(v_j w_j) = j + 9p + 1$$
$$g(w_j t_j) = j + 5p + 1$$
$$g(t_j v_{j+1}) = j + 10p + 1$$
$$g(v_j v_{j+1}) = 3j$$
$$g(u_j y_j) = j + 15p + 1$$
$$g(y_j v_{j+1}) = j + 8p + 1 \ for\ 1 \le i \le p.$$

Theorem 4.5. *Zig-Zag graph $Z(T_p)$ is a 3 -constrained total labeled graph.*

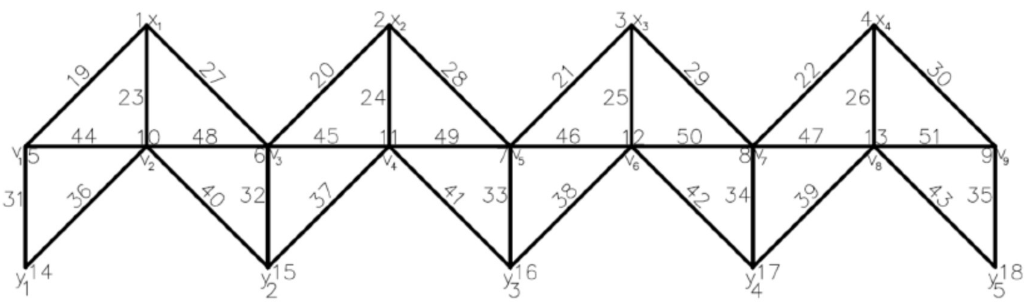

Figure 5: 3 constrained total labeling of Zig-Zag graph *Z(T*$_n$*)*

Proof. Zig-Zag graph consists of $V = \{x_j : 1 \le j \le \frac{p-1}{2}\} \cup \{y_j : 1 \le j \le \frac{p+1}{2}\} \cup \{y_j : 1 \le j \le p\}$, $E = \{x_j v_{2j+1},\ v_{2j-1} x_j,\ x_j v_{2j},\ y_j v_{2j},\ y_{j+1} v_{2j} : 1 \le j \le \frac{p-1}{2}\} \cup \{v_{2j-1} y_j : 1 \le j \le \frac{p+1}{2}\} \cup \{v_j v_{j+1} : 1 \le j \le p - 1\}$ *with* $|V| = 2p$ *and* $|E| = 4p - 3$. Now let us define a total labeling $g : V UE \to \{1, 2, . . . 6p - 3\}$ on vertices of $Z(T_p)$ *as*

$$\left.\begin{aligned} g(v_{2j}) &= j + 2 \\ g(x_j) &= j \end{aligned}\right\} 1 \le j \le \frac{p-1}{2}$$

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$$\left.\begin{aligned} g(v_{2j-1}) &= j+\frac{p-1}{2} \\ g(y_j) &= j+\frac{3p-1}{2} \end{aligned}\right\} 1 \le j \le \frac{p+1}{2}$$

Edges are labeled as

$$g(v_{2j}y_j) = j+\frac{7p-3}{2} \text{ for } 1 \le j \le \frac{p+1}{2}$$

$$\left.\begin{aligned} g(x_j v_{2j+1}) &= j+3p-1 \\ g(v_{2j-1}x_j) &= j+2p \\ g(x_j v_{2j}) &= j+\frac{5p-1}{2} \\ g(v_{2j-1}v_{2j}) &= j+5p-2 \\ g(y_j v_{2j}) &= j+4p-1 \\ g(y_{j+1}v_{2j}) &= j+\frac{9p-3}{2} \\ g(v_{2j}v_{2j+1}) &= j+\frac{11p-5}{2} \end{aligned}\right\} 1 \le j \le p$$

Theorem 4.6. Double squared chain graph *DSC* for p ≤ 3 is a 3 -constrained total labeled graph.

Proof. Let $V = \{v_j : 1 \le j \le p\} \cup \{u_j, w_j, x_j, y_j : 1 \le j \le p-1\}$ and $E = \{v_ju_j, u_jv_{j+1}, v_jw_j, w_jv_{j+1}, v_jx_j, x_jv_{j+1}, v_jy_j, y_jv_{j+1} : 1 \le j \le p-1\}$ for Double squared chain graph respectively where $|V| = 5p - 4$ and $|E| = 8p - 8$. Define a total labeling $g : V UE \to \{1, 2, \ldots, 13p-12\}$ on vertices of *DSC* as

$$g(v_j) = j + 4p - 4 \text{ for } 1 \le j \le p$$

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$$\left.\begin{aligned} g(v_j u_j) &= j + p - 1 \\ g(u_j v_{j+1}) &= j + 3p - 3 \\ g(v_j w_j) &= j + 5p - 4 \\ g(w_j v_{j+1}) &= j + 6p - 5 \\ g(v_j x_j) &= j + 7p - 6 \\ g(x_j v_{j+1}) &= j + 8p - 7 \\ g(v_j y_j) &= j + 10p - 9 \\ g(y_j v_{j+1}) &= j + 11p - 10 \end{aligned}\right\} 1 \le j \le p - 1$$

$$\left.\begin{aligned} g(u_j) &= j \\ g(w_j) &= j + 2p - 2 \\ g(x_j) &= j + 9p - 8 \\ g(y_j) &= j + 12p - 11 \end{aligned}\right\} 1 \le j \le p - 1$$

Conclusion

In this paper, we dealt with 3 constrained total labeling and defined some classic graphs. Then we obtained 3 constrained total labeling for cycle and snake related graphs.

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