ISSN:2153-182X, E-ISSN: 2153-1838
Vol. 17 No. 02 (2023)

ANALYSIS OF OPTIMIZING EFFICIENCY IN LARGE ORTHOGONAL EXPERIMENTAL DESIGNS

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Abstract

In engineering, combinatorial designs are especially important during the parameter design phase. With a comparison to Taguchi's designs, we concentrated on constructing and using experimental designs using orthogonal arrays of strength three. The methodologies used for constructing the designs were highlighted in our case study, which served as an example of the ideas underpinning experimental design.

In this case, the strain-gauge transducer's displacement signal was being measured by means of a passive filter network. The filter's transfer function was developed using Kirchhoff's current law, and equations were obtained to use in setting the network's descriptor parameters (DP) such that it would match the functional requirements. Pole location and filter gain were two functional requirements that could not be accomplished without these defining parameters.

Introduction

A lot of focus has been placed on the orthogonal design method (ODM). In both theoretical and practical situations, this method is examined in the literature. The ODM takes a few of randomly chosen locations from a vast search area. After that, a possible excellent answer is summarized statistically. Chemical and biological sciences, image processing, laser polishing, software testing procedures, algorithms, semiconductor manufacturing, optics, and resilient design are just some of the many places this method may be used. Mixed-level orthogonal array research is one example of recent theory research on the ODM method.

The existing practice of designing antennas by hand is limiting in its potential to generate new and improved antenna designs since it needs extensive subject expertise and experience, as well as being time and labor expensive. An antenna engineer will choose a class of antennas and then spend a significant amount of time testing and refining a design, often through simulation using electromagnetic modelling software. Evolutionary antenna optimization has been the subject of research since the early 1990s. An antenna may be optimized in a number of ways; some common ones are particle swarm optimization, differential evolution, and genetic algorithms/evolutionary algorithms.

An evolutionary algorithm often replaces time-consuming electromagnetic simulations throughout the antenna design process. Fast computation and imperfect simulation are then employed to cut down on execution time. One can only refer to the end outcome of such evolution

ISSN:2153-182X, E-ISSN: 2153-1838
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as a loosely developed antenna. This work is concerned with the local enhancement of rough antennas. Before constructing a goal based on the antenna's performance, we first relax its geometrical parameters within particular regions. Then, we take a modest number of antenna samples over the calmed space to identify a good one. The acronym stands for "orthogonal design method."

Methodology

In the context of experimental design and optimization, the term "orthogonal" holds distinct meanings, depending on the perspective. For an Orthogonal Array (OA), the concept of "orthogonal" refers to achieving a particular balance in a combination of factors that remains consistent regardless of how the levels of those factors are coded. This balance is crucial for experimentation and analysis. When discussing orthogonal columns, the term pertains to a geometric form of orthogonality within a multi-dimensional space. This type of orthogonality can be quantified by measuring the correlation between columns. If the correlation between two columns is zero, they are considered geometrically orthogonal.

Importantly, combinatorial orthogonality inherently implies geometric orthogonality, meaning that when factors are balanced in an OA, they also exhibit the property of geometric orthogonality. However, the reverse isn't always true—geometric orthogonality doesn't necessarily guarantee combinatorial orthogonality. Geometric orthogonality primarily applies to quantitative experimental variables and is heavily influenced by how levels of factors are coded.

In literature, discussions about Latin Hypercube Designs (LHDs) that possess orthogonal columns have taken place. An example of this can be found in Ye's work from 1998. The advantage of achieving geometric orthogonality is observed in the context of simple main effect linear regression. In such scenarios, the estimated coefficients remain unchanged, whether additional columns are included in the model or not. This stability in estimates is a valuable property for experimental design.

Expanding upon the notion, the word "3-orthogonality," as initially proposed by Sitter, Bingham and Tang in 2009, represents a heightened level of robustness in the context of orthogonality. Within the domain of 3-orthogonal arrays, it is seen that columns possess orthogonal associations not only with one another and a fixed column, but also preserve orthogonality with the products of pairs of other columns and the squares of other columns. This characteristic guarantees that estimations of primary effects remain unaffected by estimations of second-order effects, which encompass pairwise column products or squared columns.

This "3-orthogonality" property is particularly significant because it prevents misleading conclusions about main effects when second-order effects are neglected. Additionally, it enhances the efficiency of estimating second-order models. To the best of our understanding, Ye's work in 1998 was among the earliest instances of proposing 3-orthogonal LHDs. Ye's approach generated designs that involved 2k or 2 (k+1) runs, featuring 2k - 2 columns.

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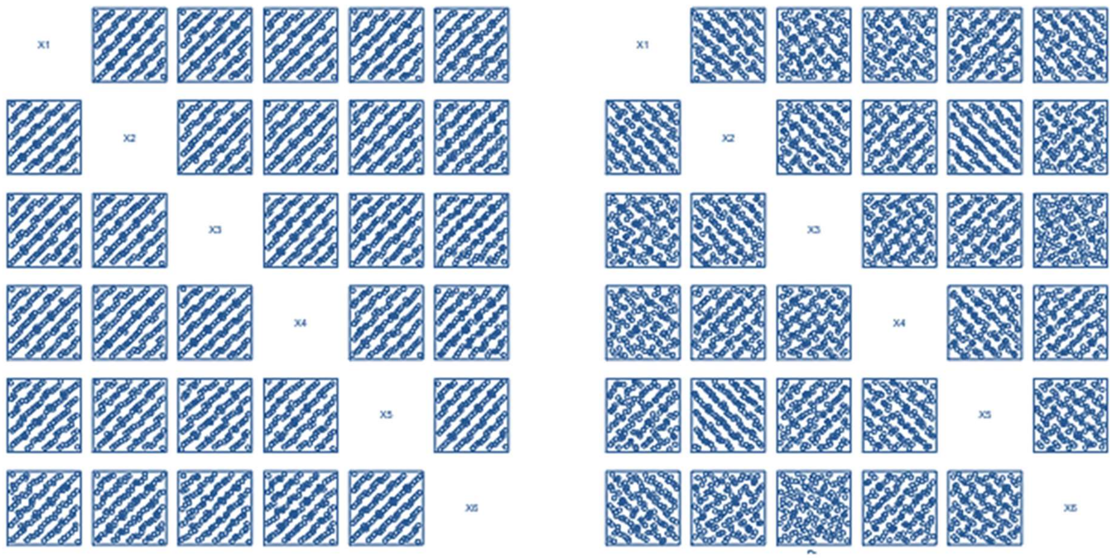

Figure 1. Enhanced 2D Representations of SOA with Six Orthogonal Columns Levels

It's important to note that while orthogonality or 3-orthogonality is a useful property, it should not be mistaken for ensuring space-filling characteristics. Therefore, it's recommended to incorporate additional criteria for space-filling, as the standard constructions can result in noticeable patterns that leave significant gaps in coverage.

For instance, let's take the example of an unoptimized orthogonal design with 125 runs. In this scenario, the arrangement of design points might appear in diagonal stripes that run parallel to each other. However, this arrangement doesn't effectively fill the space as desired. To address this, even a single round of optimization aimed at reducing the value of φp (a specific metric) can lead to a significant improvement in this behavior. This improvement is visually evident when observing the 2D projections shown in Figure 1, and by comparing the φp values. The value of φp is reduced from 0.0395 to 0.013 through the optimization process.

Nevertheless, it's important to highlight that the figure on the right-hand side also highlights the presence of systematic gaps in several of the projections. This indicates that while optimization can enhance the space-filling characteristics, it might not completely eliminate all the gaps in coverage. This underlines the need for a balanced approach that considers both orthogonality and additional criteria to ensure effective space filling and minimize patterns that leave areas unfilled.

In addition to performing a comparison with Taguchi's designs, our major focus was on constructing and applying experimental designs using the orthogonal array technique. Combinatorial designs, particularly those used in engineering design, and more specifically parameter design, were the focus of our study.

To kick off our investigation, we provided a case study as background material. To further clarify the concept of experimental design, this case study was used as an example. We wanted to

ISSN:2153-182X, E-ISSN: 2153-1838
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demonstrate the technique we used to create the experimental designs and the thinking behind our decisions through this case study.

The displacement signal from a strain-gauge transducer is measured using a passive filter network design. The filter is designed to remove the carrier frequency from the signal in order to improve its quality. Two functional requirements (FR) must be met: FR1 calls for the filter pole to be set at 6.84 Hz to reduce output distortion, and FR2 calls for the filter gain to be adjusted to achieve full-scale beam detection at a distance of 3 in.

In order to meet the functional requirements, it is necessary to ascertain the appropriate set of descriptor parameters (DP) or control variables for the passive filter network. The calculation of the filter's transfer function in the given extract lacks the inclusion of the Kirchhoff current law. To begin the design methodology, it is necessary to first derive the transfer function.

Let:

- Let V_in be the input voltage (signal from strain-gauge transducer).
- Let V_out be the output voltage (filtered signal).
- Let s be the complex frequency variable ($s = \sigma + j\omega$, where σ is the real part and j is the imaginary unit).

The transfer function of the passive filter network can be represented as H(s) = V_out / V_in.

Now, let's proceed to derive the transfer function using the Kirchhoff current law and the information given in the excerpt.

Assumptions:

- The passive filter network is a simple first-order high-pass filter.
- The filter contains a resistor (R) and a capacitor (C).

The Kirchhoff current law at the input node of the filter gives us the following equation: (V_in - V_R) / R + (V_in - V_C) / Z_C = 0

Where:

- V_R is the voltage across the resistor R.
- V_C is the voltage across the capacitor C.
- Z_C is the impedance of the capacitor, given by Z_C = 1 / (jωC).

Equation in terms of V_out and V_in:
(V_in - V_out) / R + (V_in - V_out) / Z_C = 0
Next, let's solve for V_out / V_in to find the transfer function H(s):
(V_in - V_out) / R + (V_in - V_out) / (1 / (jωC)) = 0
Now, let's simplify the equation:

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(V_in - V_out) / R + jωC (V_in - V_out) = 0
Now, factor out (V_in - V_out):
(V_in - V_out) * (1 / R + jωC) = 0
Divide both sides by (1 / R + jωC):
V_out / V_in = 1 / (1 / R + jωC)
To meet the functional requirement FR1, we need to set the pole of the filter at 6.84 Hz. The pole of a first-order high-pass filter is given by the reciprocal of the product of the resistor and capacitor values (RC):
Pole = 1 / (RC)
We want the pole to be at 6.84 Hz:
6.84 = 1 / (RC)
To meet the functional requirement FR2, we need to adjust the filter
Gain = R
Now, we have two equations to determine the values of R and C:
6.84 = 1 / (RC)
Gain = R = ±3 in
Solving these two equations will give us the values of R and C for the passive filter network.

$$\frac{V_0}{V_1} = \frac{R_g R_3}{(R_2 + R_g)(R_S + R_3) + R_S R_3 + (R_2 + R_g) R_3 R_S C}$$

$$\omega_c = \frac{(R_2 + R_g)(R_S + R_3) + R_S R_3}{2\pi (R_2 + R_g) R_3 R_S C}$$

$$D = \frac{|V_0|}{G_{\text{sen}}} = \frac{|V_S| R_S R_g}{G_{\text{sen}} \left[(R_2 + R_g)(R_S + R_3) + R_S R_3\right]}$$

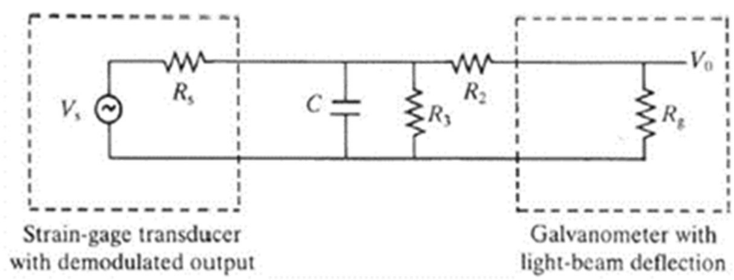

Figure 1. A passive filter network

$$E(\mu^2) = (nm^2 - V)/n$$

$$E(\sigma^2) = V = \left(\sum y^2 - S_m\right)/(n-1)$$

where $S_m = (\sum y)^2/n$.

The SN ratio is given by:

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$$\eta = 10\log_{10} E(\mu^2/\sigma^2) = 10\log_{10}[(S_{-} - V)/nV]$$

$$\omega^G = \{\omega^g \mid g \in G\}$$

$$\mathrm{Stab}_G(\omega) = \{g \in G \mid \omega^g = \omega\}$$

There is an action homomorphism ϕ: G S → Ω. The procedure involves computing orbits and stabilizers, finding representative arrays for each orbit, testing whether or not two arrays in belong to the same orbit (homomorphism), and storing the representative arrays together with the automorphism group size.

The backtrack search technique is employed to systematically search for permutations that satisfy particular requirements for orthogonal arrays, which are combinatorial arrays. Until all solutions are identified or a predetermined condition (of strength three) is violated, the backtrack search method iteratively enumerates vectors (rows) in lexicographic order, constructing arrays.

The method involves first encoding the array as a colored graph, then decoding it to a fraction after the canonical labeling graph has been found. Instead of comparing the matching arrays, this method checks for isomorphism between the colored graphs.

The steps involved in this method are as follows: first, an array is translated into a colored graph; second, the canonical graph of the colored graph is located; and third, the array is demerged from the canonical graph.

Result

When compared to Taguchi's method, our approach stood out for how easily it could be implemented. Our designs also had the distinct quality of being extensible, which meant that other combinations of factor levels could be used within the same design.

However, some of the supplementary discoveries and difficulties we faced during our research deserve mention. First, we discovered that, beyond Hedayat's method and the alternatives offered by Schoen, Murray, and Man, there is scant information concerning computational techniques for constructing orthogonal arrays (the combinatorial form). Existing techniques rely on backtrack search, which has its own set of limitations.

Second, we intend to evaluate the performance of mixed arrays design, which looks to be a strong competitor in this sector, in order to provide a thorough review.

We have pinpointed certain crucial areas for enhancement in order to overcome some anticipated drawbacks and limitations of our designs. These include deciding which interactions to measure, finding representative elements inside each orbit, figuring out the size of the arrays, selecting an appropriate design within non-isomorphic arrays, and making sure there is enough computational capacity to execute the algorithms efficiently.

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Conclusion

In conclusion, our study successfully explored the construction and application of experimental designs using orthogonal arrays in parameter design. Our innovative approach showed promising results, offering advantages in flexibility and simplicity of implementation. However, further research and refinement are necessary to overcome existing limitations and fully realize the potential of combinatorial designs in engineering applications. Overall, this study contributes valuable insights to the field of experimental design and opens avenues for future advancements in combinatorial design techniques.

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