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DATA MODELING USING TRANSMUTED DISTRIBUTIONS

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Abstract :
We discussed two distributions (the Lindley distribution and the converted Lindley distribution) and we dealt with some functions such as the pdf, cdf function, and we also discussed some statistical properties such as moments, the moment generation function, the coefficient of variation, flattening, torsion, and ordered statistics. IMSE The research concluded the importance of the converted Lindley distribution on the Lindley distribution and on all sample sizes in the practical aspect experimental .

Keywords: Lindley distribution ; Transmuted Lindley distribution; Probability Density Function ; Moments; Maximum likelihood estimation

1- Introduction:

Recently, several families of transformed distributions have been proposed, which are extended models with one or more parameters to the basic model of continuous distributions, both (Shaw and Buckley) used the rank transformation method (RTM), which is looking for a tool to build new families of non-distributives Gaussian, as it was used to modify a specific fundamental distribution for the purposes of modulating moments, especially skewness, torsion and kurtosis, introduced the QRTM quadratic rank transformation map, which many authors have used to introduce new important distributions such as the Lindley distribution transformed by (Nerovci) (2007) and the Lomax distribution Transformed by Lomax (1954), Where these new families provide more flexibility in modeling and analyzing real-life data in many applied fields, and this is why the statistical literature contains a good number of new families, where the distributions could not model some phenomena, and that is why they found shifting of the distributions in general or extending these distributions so that they may be compatible with some medical or biological phenomenon, In this article we have taken the Lindley distribution and used the transform map approach to define a new model named the Transformed Lindley Distribution (TLind), and according to the quadratic Rank transform map (QRTM). Approach the cumulative distribution function (CDF) satisfy the relationship

$$F_1(x)=(1+\lambda)F_2(x)-\lambda(F_2(x))^2 \quad (1)$$

which on differentiation yields,

$$f_1(x) = f_2(x)[(1+\lambda) - 2\lambda F_2(x)] \quad (2)$$

where $F_2(x)$ is the CDF of the base distribution

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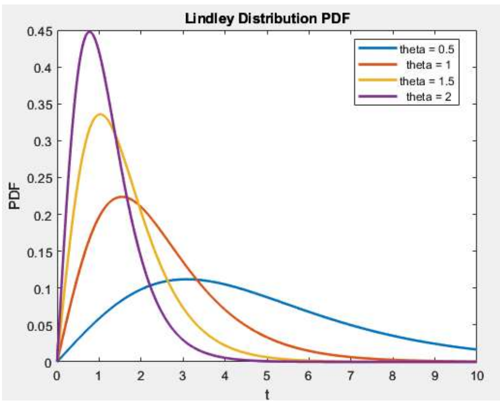
2-Transmuted Lindley Distribution ($TLind$)

The Lindley distribution describes the lifetimes of the processes and devices across a range of fields such as biology, engneering, and medicine. In fact, it's proven to be particularly effective in modeling mortality studies [3].

The (PDF) of Lindley Distribution is given by [2]:

$$f(x;\ \gamma) = \frac{\gamma^2}{\gamma+1}(1+x)e^{-\gamma x} \qquad ,\gamma > 0\,, x > 0 \qquad (3)$$

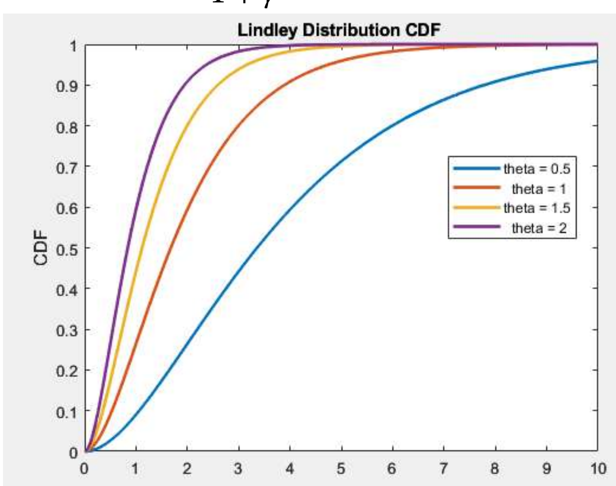
where γ is shape parameter



where γ is shape parameter

The (CDF) of Lindley Distribution is given by[2]

$$F(x;\gamma) = 1 - \frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x} \qquad ,\gamma > 0\,, x > 0 \qquad (4)$$



From equation (1) and (4) we conclude the (CDF) of the $TLind$ [4].

$$F(x) = (1 - \frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x})\ (1 + \lambda\frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x}) \qquad (5)$$

Where $|\lambda| \leq 1\,, \gamma > 0, x > 0$

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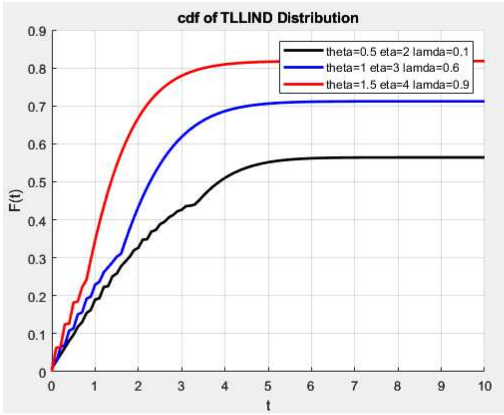
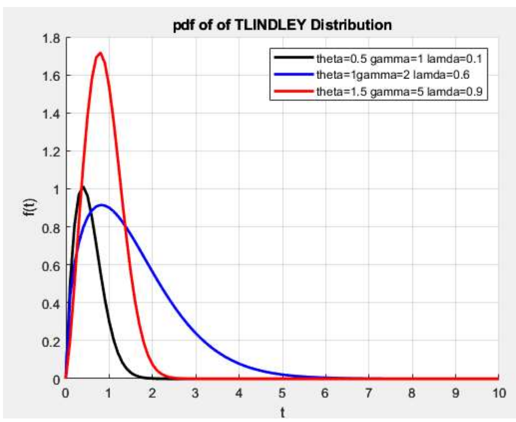


The (PDF) of (TLind) random variable may by written as [4] :

$$f(x) = \frac{\gamma^2}{1+\gamma}(1+x)e^{-\gamma}\ \left(1-\lambda+2\lambda\frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x}\right) \qquad (6)$$

Where $|\lambda| \leq 1\ \ , \gamma > 0, x > 0$



3-Mathematical Properties of ($\boldsymbol{TLind}$)

3.1 **The Moments**

Let x be a random varable that distributes the transmuted Lindley distribution that has the moment with parameter λ and θ[7]:

$$\acute{\mu}_r = E(X^r) = \int_{-\infty}^{\infty} x^r f(x)\, dx$$

$$E(X^r) = \int_0^{\infty} x^{\mathrm{r}} \frac{\gamma^2}{1+\gamma}(1+x)\mathrm{e}^{-\gamma x}(1-\lambda+2\lambda\frac{(1+\gamma+\gamma x)}{1+\gamma}\mathrm{e}^{-\gamma x})$$

$$E(X^r) = \frac{r!}{\gamma^r(\gamma+1)}\left[(1-\lambda)(\gamma+r+1)+\frac{\lambda\gamma}{2^{r-1}(\gamma+1)}(2\gamma+3+r)\right] \qquad (7)$$

The first moment of the TLind , when r =1 , we get the following relationship :

$$\mathrm{E(X)} = \mu = \frac{1}{\gamma(\gamma+1)}[(1-\lambda)(\gamma+2)+\frac{\lambda\gamma}{(\gamma+1)}(2\gamma+4)]$$

$$\mathrm{E(X)} = \frac{1}{\gamma(\gamma+1)}\left[(1-\lambda)(\gamma+2)+\frac{2\lambda\gamma}{(\gamma+1)}(\gamma+2)\right] \qquad (8)$$

when r = 2 we get the second moment of the TLind .

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$$E(X^2) = \frac{2}{\gamma^2(\gamma+1)}[(1-\lambda)(\gamma+3) + \frac{\gamma\lambda(2\gamma+5)}{2(\gamma+1)}] \quad (9)$$

The variance of the transmuted Lindley distribution is given by :

$$\mathrm{Var}(x) = E(X^2) - (E(X))^2 = \frac{2}{\gamma^2(\gamma+1)}\left[(1-\lambda)(\gamma+3) + \frac{\lambda\gamma(2\gamma+5)}{2(\gamma+1)}\right] - [\frac{1}{\gamma(\gamma+1)}((1-\lambda)(\gamma+2) + \frac{2\lambda}{\gamma+1}(\gamma+2))]^2 \quad (10)$$

The Third moment of the TLind can be obtained by taking r = 3

$$\mathrm{E}(\mathrm{X}^3) = \frac{6}{\gamma^3(\gamma+1)}[(1-\lambda)(\gamma+4) + \frac{\gamma\lambda(\gamma+3)}{2(\gamma+1)}] \quad (11)$$

The Coefficient of Variation (CV) of transmuted Lindley distribution can be obtained by the following relationship :

$$CV = \frac{\sqrt{Var(x)}}{E(X)} = \frac{\sqrt{\frac{2}{\gamma^2(\gamma+1)}\left[(1-\lambda)(\gamma+3)+\frac{\lambda\gamma(2\gamma+5)}{2(\gamma+1)}\right]-[\frac{1}{\gamma(\gamma+1)}((1-\lambda)(\gamma-2)+\frac{2\gamma\lambda(\gamma+2)}{(\gamma+1)})]^2}}{\frac{1}{\gamma(\gamma+1)}[(1-\lambda)(\gamma+2)+\frac{2\lambda\gamma}{(\gamma+1)}(\gamma+2)]} \quad (12)$$

when r = 4 , the fourth moment of transmuted Lindley distribution is given :

$$\mathrm{E}(\mathrm{X}^4) = \frac{24}{\gamma^4(\gamma+1)}\left[(1-\lambda)(\gamma+5) + \frac{\gamma\lambda(2\gamma+7)}{8(\gamma+1)}\right] \quad (13)$$

As for the coefficient of Skewness (CS) and Kurtosis (CK) of the transmuted Lindley distribution can be obtained from :

$$CS = \frac{E(X-\mu)^3}{(Var(x))^{3/2}}$$

$$E[(X-\mu)^3] = E[\sum_{i=o}^{3}\binom{3}{i}(-\mu)^i X^{3-i}]$$

$$= E\left[\binom{3}{0}(-\mu)^0X^3 + \binom{3}{1}(-\mu)X^2 + \binom{3}{2}(-\mu)^2X + \binom{3}{3}(-\mu)^3X^0\right]$$

$$= E[X^3 - 3\mu X^2 + 3\mu^2 X - \mu^3]$$

$$= E[X^3] - 3\mu E[X^2] + 3\mu^2 E[X] - \mu^3$$

$$= \acute{\mu}_3 - 3\mu\,\acute{\mu}_2 + 3\mu^2\mu - \mu^3$$

$$= \acute{\mu}_3 - 3\mu\,\acute{\mu}_2 + 2\mu^3$$

$$\mathrm{Sk} = \frac{\acute{\mu}_3 - 3\mu\,\acute{\mu}_2 + 2\mu^3}{(Var(x))^{3/2}} \quad (14)$$

As for kurtosis of TLind , it is written as follows :

$$CK = \frac{E(X-\mu)^4}{(Var(x))^2}$$

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$$E[(X-\mu)^4] = E[\sum_{i=o}^{4}\binom{4}{i}(-\mu)^i X^{4-i}]$$

$$=E\left[\binom{4}{0}(-\mu)^0X^4 + \binom{4}{1}(-\mu)X^3 + \binom{4}{2}(-\mu)^2X^2 + \binom{4}{3}(-\mu)^3X^1 + \binom{4}{4}(-\mu)^4X^0\right]$$

$$= E[X^4 - 4\mu X^3 + 6\mu^2X^2 - 4\mu^3X + \mu^4]$$

$$=E[X^4] - 4\mu E[X^3]+6\,\mu^2E[X^2] - 4\mu^3E[X] + \mu^4$$

$$= \acute{\mu}_4 \; - 4\mu\,\acute{\mu}_3 + 6\mu^2\mu_2 - 4\mu^3\mu + \mu^4$$

$$= \acute{\mu}_4 \; - 4\mu\,\acute{\mu}_3 + 6\mu^2\mu_2 - 3\mu^4$$

$$\text{CK} \quad = \frac{\acute{\mu}_4 - 4\mu\,\acute{\mu}_3 + 6\mu^2\mu_2 - 3\mu^4}{(Var(x))^2} \qquad (15)$$

3.2 Moment Generating Function

The moment generation function (MGF) of the $TLind$ is [6] :

$M_X(t) = E[e^{xt}]$

$$M_X(t) = \int_0^\infty e^{tx} f(x)dx$$

where$f(x)$ is (PDF) of transmuted Lindley distribution.

$$\text{M}_\text{X}(\text{t}) = \int_0^\infty \text{e}^{\text{t}x}\,\frac{\gamma^2}{1+\gamma}(1\;+\;x)\,\text{e}^{-\gamma x}(1 - \lambda\;+\;2\lambda\;\frac{(1+\gamma+\gamma x)}{1+\gamma}\;\text{e}^{-\gamma x}\,\text{dx}$$

$$\text{M}_\text{X}(\text{t}) = \frac{\gamma^2}{1+\gamma}\int_0^\infty \text{e}^{(\text{t}-\gamma)x}(1+x)(1-\lambda\;\;+2\lambda\frac{(1+\gamma+\gamma x)}{1+\gamma}\text{e}^{-\gamma x}\,\text{dx} \qquad (16)$$

Where $x > 0\,,\gamma > 0, |\lambda| \leq 1$

3.3 Survival Function

The survival function $S(x)$ for the TLind is given by :

$S(x) = 1 - F(x)$

$$S(x) = 1 -[(1 - \frac{(1+\gamma+\gamma\;)}{1+\gamma}\text{e}^{-\gamma x})\;(1 + \lambda\frac{(1+\gamma+\gamma\;)}{1+\gamma}\text{e}^{-\gamma x})]$$

$$S(x) = \frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x}(1-\lambda+\lambda\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma}\;) \qquad x > 0, \gamma > 0\,,\lambda > 0 \qquad (17)$$

3.4 Order Statistics

Let $X_1, X_2, ..., X_n$ are a simple random variables with distribution function (CDF)$F_X(x)$ and (PDF)$f_X(x)$.The corresponding order statistics are the X_i arranged in ascending order. The largest of the X_i is denoted by $X_{(n)}$ and the smallest of the X_i is denoted by $X_{(1)}$ [4] , the PDF of $X_{(i)}$ is given by[1]:

$$f_{X(i)}(x) = \frac{n\,!}{(i-1)!(n-i)!}f_X(x)[F_X(x)]^{i-1}[1 - F_X(x)]^{n-i} \qquad (18)$$

The PDF of order statistics for the TLind is given by[7] :

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$$f_{X(i)}(x) = \frac{n!}{(i-1)!\,(n-i)!}\left(\frac{\gamma^2}{\gamma+1}(1+x)e^{-\gamma x}(1-\lambda+2\lambda\frac{1+\gamma+\gamma x}{\gamma+1}e^{-\gamma x})\right)[(1-\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x})\,(1+\lambda\frac{1+\gamma+\gamma x}{1+\gamma}e^{-x})]^{i-1}[\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x}(1-\lambda+\lambda\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x})]^{n-i} \quad (19)$$

As for The PDF of the largest order statistic $X_{(n)}$ is given by[7]:

$$f_{X(n)}(x) = \frac{n\gamma^2}{\gamma+1}(1+x)e^{-\gamma x}(1-\lambda+2\lambda\frac{(1+\gamma+\gamma\ )}{1+\gamma}e^{-\gamma x})[\left(1-\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x}\right)\left(1+\lambda\frac{1+\gamma+\gamma}{1+\gamma}e^{-\gamma x}\right)]^{\,n-1} \quad (20)$$

And the PDF of the smallest order statistic $X_{(1)}$ is given by[7]:

$$f_{X(1)}(x) = \frac{n\gamma^2}{1+\gamma}(1+x)e^{-\gamma x}(1-\lambda+2\lambda\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x})[1-(1-\frac{1+\gamma+\gamma x}{1+\gamma}e^{-x}\gamma)\,(1+\lambda\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x}))]^{\,n-1} \quad (21)$$

4-Estimation Method of ($TLind$)

We used the Maximum Likelihood Method for estimating the parameters of TLind . Let $X_1, X_2, \dots, X_n$ is the sample size n at random from the $TLind$ .Then the likelihood function You write as follows [7]:

$$L = (x_i, \gamma, \lambda) = \prod_{i=1}^{n} f(x_i, \gamma, \lambda)$$

$$L = \prod_{i=1}^{n}\left[\frac{\gamma^2}{\gamma+1}(1+x)e^{-\gamma x}\left(1-\lambda+2\lambda\frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma}\ \right)\right]$$

$$L = \frac{\gamma^{2n}}{(\gamma+1)^n}e^{-\sum_{i=1}^{n}\gamma x_i}\prod_{i=1}^{n}(1+x_i).(1-\ \lambda+2\lambda\frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x})$$

$$lnL = 2n\,ln\gamma - nln(\gamma+1) - \sum_{i=1}^{\gamma n} x_i + \sum_{=1}^{n}\ln(1+x_i) + \sum_{i=1}^{n}\ln(1-\lambda+2\lambda\frac{1+\gamma+\gamma x_i}{\gamma+1}\ e^{-\gamma x_i}$$

$$\frac{\partial L}{\partial \gamma} = \frac{2n}{\gamma} - \frac{n}{\gamma+1} - \sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\frac{2\lambda x_i\frac{1+\gamma+\gamma x_i}{1+\gamma}\ e^{-\gamma x_i}e^{-\gamma x_i}\left[\frac{1}{(\gamma+1)^2}-\frac{1+\gamma+\gamma x_i}{1+\gamma}\right]}{1-\ \lambda+2\lambda\frac{1+\gamma+\gamma x_i}{1+\gamma}e^{-\gamma x_i}} = 0$$

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$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} \frac{2\frac{1+\gamma+\gamma x_i}{\gamma+1} e^{-\gamma x_i} - 1}{1-\lambda+2\lambda\frac{1+\gamma+\gamma x_i}{\gamma+1} e^{-\gamma x_i}} = 0$$

And using quasi-Newtonian algorithm to maximize the incoming registration probability functio

5-Simulation Concept

We conducted Monte Carlo simulation studies to assess on the finite sample behavior of the maximum likelihood and the simulations were carried out using the statistical software Matlab . 2022b program The true parameter values used in the data generating processes are $\gamma = 2, \lambda = 0.1$and sample sizes n=25 ,50,75,100,150.

Assuming that the default values for the parameters of Lind is $\gamma = 2$ and TLind is $\gamma = 2$, $\lambda = 0.1$, the estimation results can be summarized as shown in Table (1).

Table (1): The values of the real density function and estimated according to the method of MLE of the distribution of Lindley, and TLindley at the theoretical samples sizes :

N	$f(x)_{Lin}$	$\hat{f}(x)_{Lin}$	MSE	$f(x)_{TL}$	$\hat{f}(x)_{TL}$	MSE
25	0.92474	0.65981	0.07019	0.89851	0.86029	0.00146
	0.73460	0.58934	0.02110	0.88173	0.84563	0.00130
	0.72927	0.58717	0.02019	0.84543	0.81357	0.00102
	0.70275	0.57618	0.01602	0.71974	0.70012	0.00038
	0.65865	0.55723	0.01029	0.69649	0.67883	0.00031
	0.56662	0.51478	0.00839	0.69580	0.67820	0.00031
	0.49406	0.47817	0.00813	0.62453	0.61245	0.00015
	0.42376	0.43956	0.00671	0.59224	0.58244	0.00010
	0.41764	0.43604	0.00269	0.53233	0.52639	0.00004
	0.17255	0.26271	0.00172	0.44151	0.44057	0.00004
IMSE			0.23936			0.00021
AIC			-12.38			-54.27134

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			310			
Best			$\hat{f}(x)_{TLind}$			
50	**0.85914**	**0.57241**	**0.08221**	**0.62546**	**0.58535**	**0.00161**
	0.82167	**0.56285**	**0.06699**	**0.54986**	**0.52551**	**0.00059**
	0.81347	**0.56069**	**0.06390**	**0.44910**	**0.44260**	**0.00044**
	0.61330	**0.50001**	**0.01283**	**0.44640**	**0.44033**	**0.00044**
	0.61005	**0.49889**	**0.01236**	**0.44190**	**0.43654**	**0.00044**
	0.60179	**0.49600**	**0.01196**	**0.44188**	**0.43652**	**0.00043**
	0.54679	**0.47593**	**0.01194**	**0.43398**	**0.42984**	**0.00043**
	0.47636	**0.44779**	**0.01181**	**0.41345**	**0.41238**	**0.00042**
	0.46208	**0.44171**	**0.01174**	**0.41073**	**0.41006**	**0.00042**
	0.44592	**0.43466**	**0.01146**	**0.39884**	**0.39986**	**0.00042**
IMSE			**0.12295**			**0.00021**
AIC			**-26.75951**			**-58.56444**
Best			$\hat{f}(x)_{TLind}$			
75	**0.73124**	**0.56055**	**0.02913**	**0.64437**	**0.67575**	**0.00098**
	0.70032	**0.54975**	**0.02267**	**0.63848**	**0.66871**	**0.00091**
	0.68559	**0.54446**	**0.01992**	**0.63165**	**0.66056**	**0.00084**
	0.67408	**0.54026**	**0.01791**	**0.62700**	**0.65502**	**0.00079**
	0.63865	**0.52697**	**0.01247**	**0.60661**	**0.63084**	**0.00059**
	0.62623	**0.52218**	**0.01083**	**0.59247**	**0.61415**	**0.00047**
	0.60947	**0.51559**	**0.01044**	**0.56376**	**0.58055**	**0.00029**

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	0.60792	0.51498	0.01042	0.54947	0.56393	0.00029
	0.59529	0.50991	0.01004	0.51868	0.52840	0.00029
	0.54711	0.48985	0.00988	0.51583	0.52513	0.00028
IMSE			0.09471			0.00017
AIC			-41.15213			-90.78444
Best			$\hat{f}(x)_{TLind}$			
100	0.70475	0.57777	0.01612	0.53566	0.55402	0.00086
	0.69550	0.57386	0.01480	0.53110	0.54827	0.00086
	0.68512	0.56942	0.01339	0.50868	0.52018	0.00085
	0.62437	0.54249	0.00834	0.49633	0.50486	0.00084
	0.62263	0.54170	0.00818	0.47764	0.48188	0.00084
	0.61222	0.53690	0.00781	0.47103	0.47380	0.00083
	0.58319	0.52325	0.00776	0.46026	0.46071	0.00083
	0.56516	0.51454	0.00707	0.45940	0.45968	0.00082
	0.55674	0.51042	0.00670	0.45799	0.45797	0.00078
	0.55403	0.50909	0.00655	0.44495	0.44226	0.00076
IMSE			0.07909			0.00017
AIC			-55.51464			-110.78886
Best			$\hat{f}(x)_{TLind}$			
150	0.57378	0.52586	0.00746	0.47742	0.47760	0.00000
	0.57299	0.52545	0.00745	0.47556	0.47574	0.00000

	0.56240	**0.51998**	**0.00744**	**0.47547**	**0.47565**	**0.00000**
	0.55617	**0.51672**	**0.00743**	**0.46693**	**0.46707**	**0.00000**
	0.53235	**0.50411**	**0.00738**	**0.45769**	**0.45780**	**0.00000**
	0.52597	**0.50067**	**0.00728**	**0.45197**	**0.45206**	**0.00000**
	0.48959	**0.48064**	**0.00726**	**0.44701**	**0.44708**	**0.00000**
	0.48049	**0.47551**	**0.00722**	**0.44577**	**0.44583**	**0.00000**
	0.46973	**0.46937**	**0.00722**	**0.42570**	**0.42568**	**0.00000**
	0.57378	**0.52586**	**0.00746**	**0.41600**	**0.41595**	**0.00000**
IMSE			**0.06023**			**0.00001**
AIC			**-84.30144**			**-212.54533**
Best			$\hat{f}(x)_{TLind}$			

Table(1)showed that under theoretical values of parameters of distributions (θ=2 of Lindley distribution and (θ=2, λ=0.1) of transmuted Lindley distribution the following:
under n=25, 50,75, 100, 150 the transmuted Lindley distribution is the best of other distributions because the IMSE and AIC for $\hat{f}(x)_{TLind}$ less than the Lindley distribution for all sizes

6- Conclusion

The efficiency of the transmuted distributions proved, as we noticed through the results that the transmuted Lindley distribution reconciled with the original distribution

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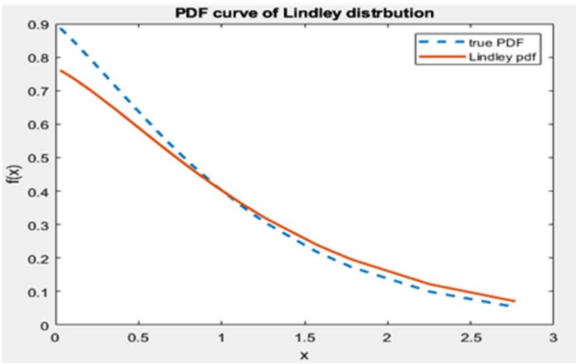

Figure (1.1) Real and Maximum Likelihood curve for Lindley distribution under n=25

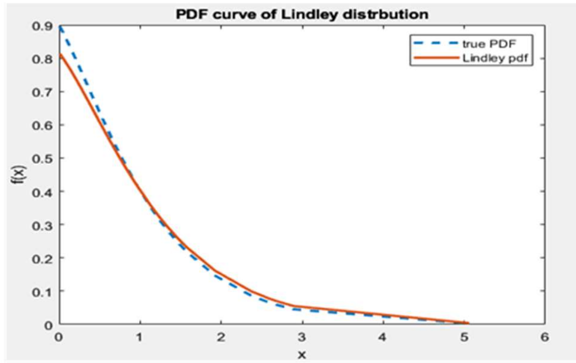

Figure (1.2) Real and Maximum Likelihood curve for Lindley distribution under n=50

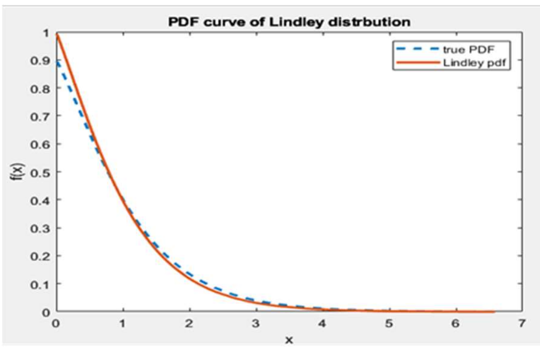

Figure (1.3) Real and Maximum Likelihood curve for Lindley distribution under n=100

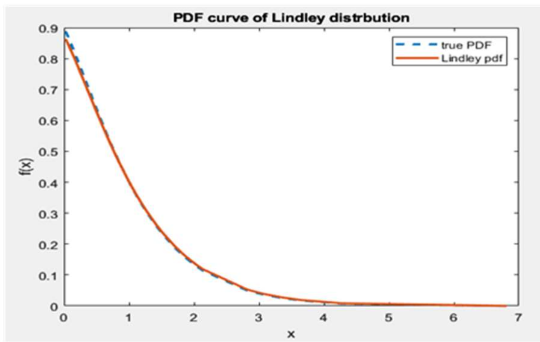

Figure (1.4) Real and Maximum Likelihood curve for Lindley distribution under n=75

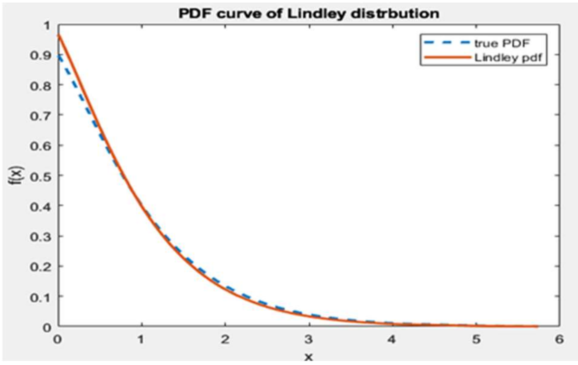

Figure (1.5) Real and Maximum Likelihood curve for Lindley distribution under n=150

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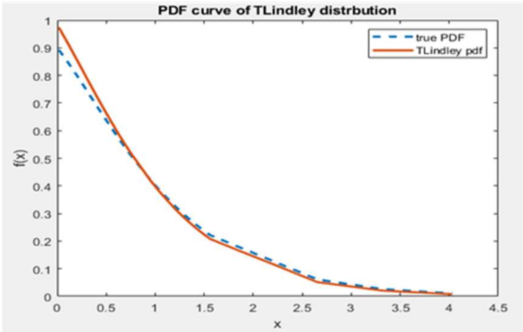

Figure (1.6) Real and Maximum Likelihood curve for TLindley distribution under n=25

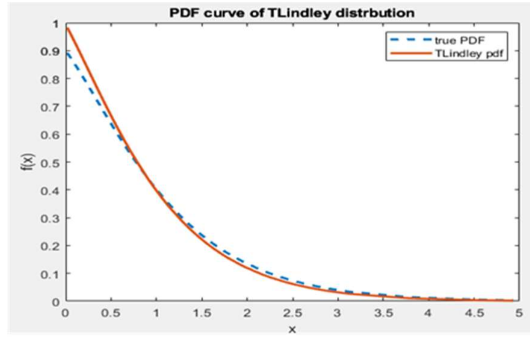

Figure (1.7) Real and Maximum Likelihood curve forTLindley distribution under n=50

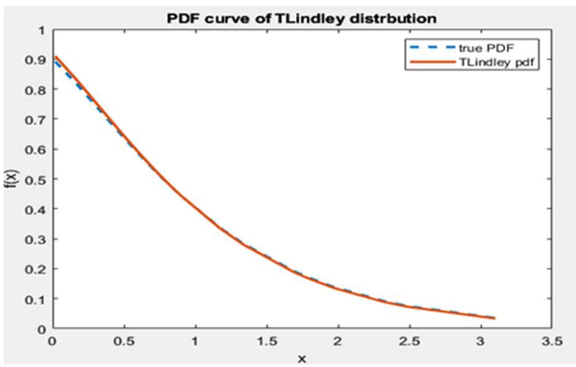

Figure (1.8) Real and Maximum Likelihood curve for TLindley distribution under n=75

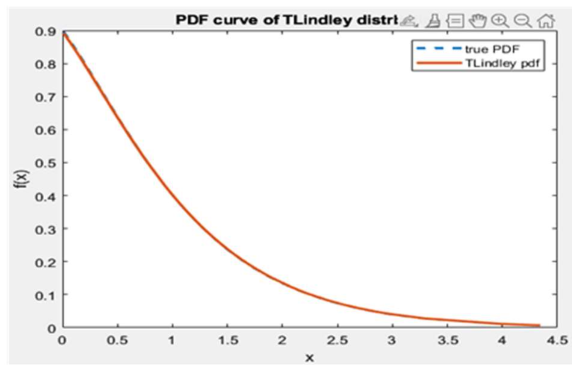

Figure (1.9) Real and Maximum Likelihood curve for TLindley distribution under n=100

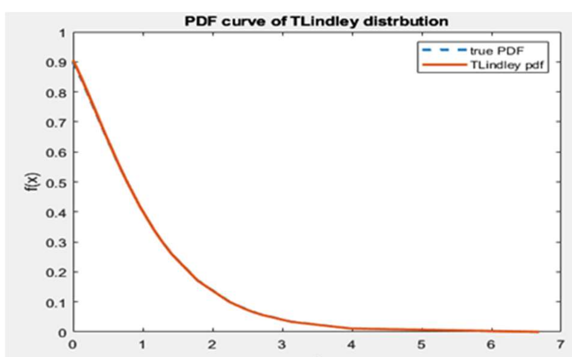

Figure (1.10) Real and Maximum Likelihood curve for TLindley distribution under n=150

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