

SOFT R- MORPHISM OF INTUITIONISTIC FUZZY SOFT HYPERGRAPHS**K. K. Myithili¹ and R.D. Beulah²**¹Department of Mathematics (CA), Vellalar College for Women, Erode-683012, Tamilnadu, India.²Department of Mathematics, Sri GVG Visalakshi College for Women, Udumalpet-642128, Tamilnadu, India.E-Mail: ¹mathsmyth@gmail.com and ²beulahrd2019@yahoo.com**Abstract**

An Intuitionistic Fuzzy Soft Set (IFSS) is a fuzzy soft set extension that deals with ambiguous information corresponding to their various parameters. The IFSS serves as a more efficient tool for dealing with uncertain data than a fuzzy soft set. Hypergraphs are used to represent almost any complex situation that involves objects and their relationships. The concept of IFSS is applied to hypergraphs, and the concept of Intuitionistic Fuzzy Soft Hypergraphs is presented (IFSHGs). We defined regular, totally regular, and perfectly regular IFSHGs and illustrated them with examples. We also investigated the isomorphism of IFSHGs and their properties. The concepts of soft R - morphism of IFSHGs and linearity of IFSHG were introduced.

Keywords: Regular, Totally Regular, Isomorphism, soft R - morphism.

1 Introduction

Molodtsov [7] pioneered the notion of soft set theory for dealing with uncertainty from the perspective of parametrization in 1999. Maji [4] et al. introduced the concept of soft set theory and expanded it to incorporate fuzzy soft sets. In 1999, Atanassov [1] introduced the noble concept of an intuitionistic fuzzy set. Maji [5] et al. then proposed an intuitionistic fuzzy soft set as an extension of a fuzzy soft set. Euler created the idea of graph theory. The notion of graphs is generalized to hypergraphs, which are composed of a set V of vertices and a collection of subsets of V . Berge [2] introduced the concept of fuzzy hypergraphs in 1976. Nagoorgani [9] introduced regular fuzzy graphs in 2009. The concepts of intuitionistic fuzzy graphs, intuitionistic fuzzy hypergraphs, and isomorphism on intuitionistic fuzzy directed hypergraphs were introduced by Parvathi [10, 11, 12] et al. Pradeepa [13] proposed regular and totally regular intuitionistic fuzzy hypergraphs later in 2018. In 2014, Thumbakara [15] and George addressed the importance of soft graphs in detail. Mohinta and Samanta [6] introduced the concept of fuzzy soft graphs in 2015. Then, several authors [3,14] developed intuitionistic fuzzy soft graphs. In 2018, Thilagavathi[16] introduced intuitionistic fuzzy soft hypergraphs (IFSHGs), which were later studied by Myithili and Beulah[8]. The concepts of Regular, Totally Regular, Perfectly Regular, and Uniform IFSHGs are illustrated with examples in this paper. Additionally, the linearity of IFSHG and soft R-morphism of IFSHGs are introduced. Isomorphism of IFDHG is studied further and examples are presented. It was further revealed that an IFSHG is regular and totally regular IFSHGs if $(S\mu, S\nu)$ is a constant function.

Notation list:

Throughout this paper, the following notations were used.

- U be the universe set and R be the set of all parameters.
- $H^* = (N, S, R)$ is an IFSHG.
- $(N \mu, N \nu)$ or simply (μ_i, ν_i) denotes the degrees of membership and non- membership of the vertex $v_i \in V$, such that $0 \leq N \mu + N \nu \leq 1$.
- $(S \mu, S \nu)$ or simply (μ_{ij}, ν_{ij}) denotes the degrees of membership and non- membership of the hyperedge $v_i, v_j \in V \times V$, such that $0 \leq S \mu + S \nu \leq 1$.
- $P(V \times V)$ is an intuitionistic fuzzy power set.
- $P(V)$ and $P(E)$ be the set of all intuitionistic fuzzy soft set over vertices V
- and hyperedges E respectively.
- The support of an Intuitionistic fuzzy soft set V in S is denoted by $\text{supp } E_j(a_i) =$
- $\{v_i / S \mu(a_i) > 0 \text{ and } S \nu(a_i) > 0, a_i \in R\}$.
- $S \mu : N \rightarrow [0, 1], S \nu : N \rightarrow [0, 1]$ is a constant function.

2 Preliminaries

The basic definitions relating to intuitionistic fuzzy set, intuitionistic fuzzy soft set, intuitionistic fuzzy hypergraphs and intuitionistic fuzzy soft hypergraphs are dealt in this section.

Definition 2.1. [1] Let a set E be fixed. An intuitionistic fuzzy set (IFS) V in E is an object of the form $V = \{(v_i, \mu_i(v_i), \nu_i(v_i)) | v_i \in E\}$, where the function $\mu_i : E \rightarrow [0, 1]$ and $\nu_i : E \rightarrow [0, 1]$ determine the degree of membership and the degree of non-membership of the element $v_i \in E$, respectively and for every $v_i \in E, 0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$.

Definition 2.2. [10] Let E be the fixed set and $V = \{(v_i, \mu_i(v_i), \nu_i(v_i)) | v_i \in V\}$ be an IFS. Six types of Cartesian product of n subsets (crisp sets) V_1, V_2, \dots, V_n of V over E are defined as follows, of V over E are defined as follows,

$$V_1 \times_1 V_2 \times_1 V_3 \dots \times_1 V_n = \{(v_1, v_2, \dots, v_n), \prod_{i=1}^n \mu_i, \prod_{i=1}^n \nu_i | v_1 \in V_1, v_2 \in V_2, \dots, v_n \in V_n\}$$

$$V_1 \times_2 V_2 \times_2 V_3 \dots \times_2 V_n = \{(v_1, v_2, \dots, v_n), \sum_{i=1}^n \mu_i - \sum_{i \neq j} \mu_i \mu_j + \sum_{i \neq j \neq k} \mu_i \mu_j \mu_k - \dots + (-1)^{n-2} \sum_{i \neq j \neq k \dots \neq n} \mu_i \mu_j \mu_k \dots \mu_n + (-1)^{n-1}, \prod_{i=1}^n \mu_i, \prod_{i=1}^n \nu_i | v_1 \in V_1, v_2 \in V_2, \dots, v_n \in V_n\}$$

$$V_1 \times_3 V_2 \times_3 V_3 \dots \times_3 V_n = \{(v_1, v_2, \dots, v_n), \sum_{i=1}^n \nu_i - \sum_{i \neq j} \nu_i \nu_j + \sum_{i \neq j \neq k} \nu_i \nu_j \nu_k - \dots + (-1)^{n-2} \sum_{i \neq j \neq k \dots \neq n} \nu_i \nu_j \nu_k \dots \nu_n + (-1)^{n-1}, \prod_{i=1}^n \mu_i, \prod_{i=1}^n \nu_i | v_1 \in V_1, v_2 \in V_2, \dots, v_n \in V_n\}$$

$$V_1 \times_4 V_2 \times_4 V_3 \dots \times_4 V_n = \{(v_1, v_2, \dots, v_n), \min(\mu_1, \dots, \mu_n), \max(\nu_1, \dots, \nu_n) | v_1 \in V_1, v_2 \in V_2, \dots, v_n \in V_n\}$$

$$V_1 \times_5 V_2 \times_5 V_3 \dots \times_5 V_n = \{(v_1, v_2, \dots, v_n), \max(\mu_1, \dots, \mu_n), \min(\nu_1, \dots, \nu_n) | v_1 \in V_1, v_2 \in V_2, \dots, v_n \in V_n\}$$

It must be noted that $v_i \times_s v_j$ is an IFS, where $s = 1, 2, 3, 4, 5, 6$.

Definition 2.3. [5] If $M \subseteq R$ and IF^U denotes the set of all intuitionistic fuzzy sets of U. A pair (F, M) is called an intuitionistic fuzzy soft set over U, where

intuitionistic fuzzy approximation function is given by $F = (F_\mu, F_\nu) : M \rightarrow \mathbf{IF}^U$.

Definition 2.4. [3, 11] An intuitionistic fuzzy soft graph (IFSG) on a nonempty set V is an ordered 3-tuple $G = (F, K, R)$ such that

- (F, R) is an intuitionistic fuzzy soft set over V .
- (K, R) is an intuitionistic fuzzy relation on V . That is $K: R \rightarrow P(V \times V)$.
- $(F(a), K(a))$ is an intuitionistic fuzzy soft subgraph, for all $a \in R$.

That is,

1. $K_\mu(a)(uv) \leq \min \{F_\mu(a)(u), K_\mu(a)(v)\}$
2. $K_\nu(a)(uv) \leq \max \{F_\nu(a)(u), K_\nu(a)(v)\}$,

such that $0 \leq K_\mu(a)(uv) + K_\nu(a)(uv) \leq 1$, for every $a \in R$ and $u, v \in V$.

Note: The fifth cartesian product has been used throughout this paper,

$$V_{i1} \times V_{i2} \times V_{i3} \times \dots \times V_{in} = \{ \langle v_1, v_2, \dots, v_n \rangle, \max(\mu_1, \mu_2, \dots, \mu_n), \min(v_1, v_2, \dots, v_n) \mid v_1 \in V_1, v_2 \in V_2, \dots, v_n \in V_n \}.$$

Definition 2.5. [8] An intuitionistic fuzzy soft hypergraphs (IFSHGs) $\tilde{H} = (H^*, \mathfrak{R}, \mathfrak{S}, R)$ is an ordered 4-tuple, such that

- $H^* = \langle V, E \rangle$ is an intuitionistic fuzzy hypergraph.
- (\mathfrak{R}, R) is an intuitionistic fuzzy soft set over V .
- (\mathfrak{S}, R) is an intuitionistic fuzzy relation on V . That is $\mathfrak{S} : R \rightarrow P(V \times V)$.
- $(\mathfrak{R}(a), \mathfrak{S}(a))$ is an intuitionistic fuzzy soft subhypergraph, for all $a \in R$.

That is,

1. $\mathfrak{S}_\mu(a)(x_1, \dots, x_n) \leq \max \{ \mathfrak{R}_\mu(a)(x_1), \mathfrak{R}_\mu(a)(x_2), \dots, \mathfrak{R}_\mu(a)(x_n) \}$
2. $\mathfrak{S}_\nu(a)(x_1, \dots, x_n) \leq \min \{ \mathfrak{R}_\nu(a)(x_1), \mathfrak{R}_\nu(a)(x_2), \dots, \mathfrak{R}_\nu(a)(x_n) \}$,

such that $0 < \mathfrak{S}_\mu(a)(x_1, \dots, x_n) + \mathfrak{S}_\nu(a)(x_1, \dots, x_n) \leq 1$, for all $a \in R$ and $x_1, \dots, x_n \in V$. Where, $\mathfrak{S}_\mu(a)(x_1, \dots, x_n)$ denotes the degree of membership and $\mathfrak{S}_\nu(a)(x_1, \dots, x_n)$ denotes the degree of non-membership of vertex to intuitionistic fuzzy soft hyperedge \mathfrak{S}_j .

Intuitionistic fuzzy soft hypergraph is denoted by $\tilde{H} = (\mathfrak{R}(a), \mathfrak{S}(a))$ or

$$\tilde{H} = \{ \tilde{H}(a_1), \tilde{H}(a_2), \dots, \tilde{H}(a_n) \}.$$

In other words, an intuitionistic fuzzy soft hypergraphs is a parameterized family of intuitionistic fuzzy hypergraphs.

Example 1. Consider an IFSHG $\tilde{H} = (H^*, \mathfrak{R}, \mathfrak{S}, R)$, such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

and $\mathfrak{S} = \{ \mathfrak{S}_1, \mathfrak{S}_2, \mathfrak{S}_3, \mathfrak{S}_4, \mathfrak{S}_5 \}$. Let $R = \{a_1, a_2\}$ be a parameter set. Let (\mathfrak{R}, R) be an intuitionistic fuzzy soft set over V with its approximate function $\mathfrak{R} : R \rightarrow P(V)$. Let (\mathfrak{S}, R) be an intuitionistic fuzzy soft set over E with its approximate function $\mathfrak{S} : R \rightarrow P(E)$.

An IFSHG $\tilde{H} = \{ \tilde{H}(a_1), \tilde{H}(a_2) \}$ is shown in fig1.

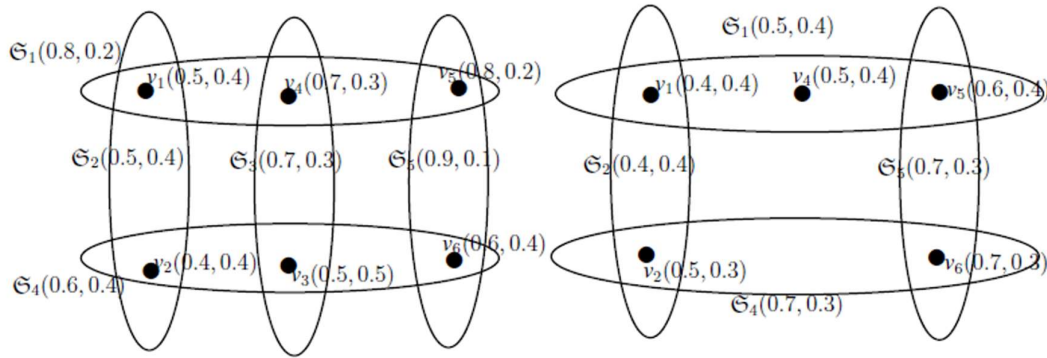


Figure 1: The IFSHG $\tilde{H} = \{ \tilde{H}(a_1), \tilde{H}(a_2) \}$ corresponding to the parameters a_1 and a_2 .

3. Regular Intuitionistic Fuzzy Soft Hypergraphs

Definition 3.1. The order of an IFSHG is

$$\mathcal{O}(\tilde{H}) = \left[\sum_{a_i \in R} \left(\sum_{v \in V} \mathfrak{R}_\mu(v) \right), \sum_{a_i \in R} \left(\sum_{v \in V} \mathfrak{R}_\nu(v) \right) \right].$$

Definition 3.2. The size of an IFSHG is

$$\mathcal{S}(\tilde{H}) = \left[\sum_{a_i \in R} \left(\sum_{v_1 \dots v_n \in \mathfrak{S}} \mathfrak{S}_\mu(a_i)(v_1 \dots v_n) \right), \sum_{a_i \in R} \left(\sum_{v_1 \dots v_n \in E} \mathfrak{S}_\nu(a_i)(v_1 \dots v_n) \right) \right].$$

Definition 3.3. The open neighborhood of a vertex $v_i(a_i)$ in the intuitionistic fuzzy soft hypergraph is denoted by $O_N(a_i)(v_1, \dots, v_n)$ and it is defined by the set of adjacent vertices of $v_i(a_i)$ excluding that vertex corresponding to the parameter.

Definition 3.4. The closed neighborhood of a vertex $v_i(a_i)$ in the intuitionistic fuzzy soft hypergraph is denoted by $C_N[(a_i)(v_1, \dots, v_n)]$ and it is defined by the set of adjacent vertices of $v_i(a_i)$ including that vertex corresponding to the parameter.

Example 2. For the above Example 1,

The open neighborhood of a vertex $v_2(a_1) = \{v_1, v_3, v_6\}$ and $v_2(a_2) = \{v_1, v_6\}$.

The closed neighborhood of a vertex $v_2(a_1) = \{v_1, v_2, v_3, v_6\}$ and $v_2(a_2) = \{v_1, v_2, v_6\}$.

Definition 3.5. If $\tilde{H} = (\mathfrak{N}, \mathfrak{S}, R)$ be an Intuitionistic Fuzzy Soft Hypergraph, then the degree of open neighborhood for a vertex $v_i(a_i)$ is denoted by $\text{deg}_{ON}(v_i(a_i))$ and it is defined by $\text{deg}_{ON}(v_i(a_i)) = \text{deg}_\mu(v_i(a_i))$, $\text{deg}_\nu(v_i(a_i))$, where $\text{deg}_\mu(v_i(a_i)) = \sum_{v_i \in \mathfrak{N}} \mathfrak{S}_\mu(v_i(a_i))$ and $\text{deg}_\nu(v_i(a_i)) = \sum_{v_i \in \mathfrak{N}} \mathfrak{S}_\nu(v_i(a_i))$.

Definition 3.6. If $\tilde{H} = (\mathfrak{N}, \mathfrak{S}, R)$ be an Intuitionistic Fuzzy Soft Hypergraph, then the degree of closed neighborhood for a vertex $v_i(a_i)$ is denoted by $\text{deg}_{CN}[v_i(a_i)]$ and it is defined by $\text{deg}_{CN}[v_i(a_i)] = (\text{deg}_\mu[v_i(a_i)], \text{deg}_\nu[v_i(a_i)])$, where $\text{deg}_\mu[v_i(a_i)] = \text{deg}_\mu(v_i(a_i)) + \mathfrak{S}_\mu(v_i(a_i))$ and $\text{deg}_\nu[v_i(a_i)] = \text{deg}_\nu(v_i(a_i)) + \mathfrak{S}_\nu(v_i(a_i))$.

Example 3. For the above Example 1, the degree of open neighborhood for a vertex $v_2(a_1) = \langle 1.1, 0.8 \rangle$ and $v_2(a_2) = \langle 1.1, 0.7 \rangle$. The degree of closed neighborhood for a vertex $v_2(a_1) = \langle 2.0, 1.7 \rangle$ and $v_2(a_2) = \langle 2.0, 1.4 \rangle$.

Definition 3.7. Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S}, R)$ be an IFSHG. If all the vertices in \mathfrak{N} have the same degree of open neighborhood degree (k_i, k'_i) for the corresponding parameters, then \tilde{H} is said to be (k_i, k'_i) - regular intuitionistic fuzzy soft hypergraph.

Remark 1. Any intuitionistic fuzzy soft hypergraphs with two vertices and one hyperedge is always a regular IFSHG.

Definition 3.8. Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S}, R)$ be an IFSHG. If all the vertices in \mathfrak{N} have the same degree of closed neighborhood degree (p_i, p'_i) for the corresponding parameters, then \tilde{H} is said to be (p_i, p'_i) - totally regular intuitionistic fuzzy soft hypergraph.

Definition 3.9. Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S}, R)$ be an IFSHG. If \tilde{H} is (k_i, k'_i) - regular and (p_i, p'_i) - totally regular intuitionistic fuzzy soft hypergraph, then it is said to be perfectly regular IFSHG.

Example 4. Consider an IFSHG $\tilde{H} = (H^*, \mathfrak{N}, \mathfrak{S}, R)$, such that $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_2v_3, v_3v_4, v_1v_4\}$. Let $R = \{a_1\}$ be a parameter set. Let (\mathfrak{N}, R) be an intuitionistic fuzzy soft set over V with its approximate function $\mathfrak{N} : R \rightarrow P(V)$. $\mathfrak{N}(a_1) = \{v_1\langle 0.7, 0.3 \rangle, v_2\langle 0.7, 0.3 \rangle, v_3\langle 0.7, 0.3 \rangle, v_4\langle 0.7, 0.3 \rangle\}$

Let (\mathfrak{S}, R) be an intuitionistic fuzzy soft set over E with its approximate function $\mathfrak{S} : R \rightarrow P(E)$.

$$\mathfrak{S}(a_1) = \{v_1v_2\langle 0.8, 0.2 \rangle, v_2v_3\langle 0.8, 0.2 \rangle, v_3v_4\langle 0.8, 0.2 \rangle, v_1v_4\langle 0.8, 0.2 \rangle\}.$$

The open neighborhood degree of the vertices for the parameter a_1 are same. That is, $deg(v_1) = deg(v_2) = deg(v_3) = deg(v_4) = \langle 1.4, 0.6 \rangle$. Hence IFSHG is said to be regular of degree $\langle 1.4, 0.6 \rangle$ (or) $\langle 1.4, 0.6 \rangle$ - regular intuitionistic fuzzy soft hypergraph.

The closed neighborhood degree of the vertices for the parameter a_1 are same. That is, $deg(v_1) = deg(v_2) = deg(v_3) = deg(v_4) = \langle 3.0, 1.0 \rangle$. Hence IFSHG is said to be totally regular of degree $\langle 2.1, 0.9 \rangle$ (or) $\langle 2.1, 0.9 \rangle$ - totally regular intuitionistic fuzzy soft hypergraph.

Remark 2. Any intuitionistic fuzzy soft hypergraphs with different membership and non-membership values need not to be regular or totally regular under parameterization.

Definition 3.10. If all the hyper-edges corresponding to their parameters have the same cardinality, then IFSHG is said to be (k_i, k'_i) - uniform IFSHG.

Example 5. Consider an IFSHG $\tilde{H} = (H^*, \mathfrak{R}, \mathfrak{S}, R)$, such that $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{v_1v_2v_3, v_3v_4v_5\}$. Let $R = \{a_1\}$ be a parameter set. Let (\mathfrak{R}, R) be an intuitionistic fuzzy soft set over V with its approximate function $\mathfrak{R} : R \rightarrow P(V)$. $\mathfrak{R}(a_1) = \{v_1\langle 0.5, 0.4 \rangle, v_2\langle 0.6, 0.3 \rangle, v_3\langle 0.7, 0.2 \rangle, v_4\langle 0.4, 0.3 \rangle, v_5\langle 0.3, 0.2 \rangle\}$

Let (\mathfrak{S}, a) be an intuitionistic fuzzy soft set over E with its approximate function $\mathfrak{S} : R \rightarrow P(E)$.

$$\mathfrak{S}(a_1) = \{v_1v_2v_3\langle 0.8, 0.2 \rangle, v_3v_4v_5\langle 0.8, 0.2 \rangle\}.$$

The $(0.8, 0.2)$ - Uniform IFSHG is shown in Fig: 2,

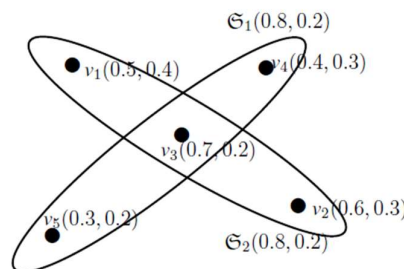


Figure 2: $(0.8, 0.2)$ - Uniform IFSHG

Definition 3.11. Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S}, R)$ be an IFSHG. For any $a_i \in R$, the intuitionistic fuzzy soft hypergraph $\tilde{H} = (\mathfrak{N}(a_i), \mathfrak{S}(a_i))$ is said to be linear if for each $\mathfrak{C}_i(a_i), \mathfrak{C}_j(a_i) \in \mathfrak{N}(a_i)$

- (i) $supp\mathfrak{C}_i(a_i) \subseteq supp\mathfrak{C}_j(a_i) \Rightarrow i = j.$
- (ii) $|supp\mathfrak{C}_i(a_i) \cap supp\mathfrak{C}_j(a_i)| \leq 1$ for all $a_i \in R.$

The intuitionistic fuzzy soft hypergraph \tilde{H} is linear if $\tilde{H}(a_i)$ is linear, for each $a_i \in R.$

Definition 3.12. Let $\tilde{H}_1 = (\mathfrak{N}_1, \mathfrak{S}_1, R_1)$ and $\tilde{H}_2 = (\mathfrak{N}_2, \mathfrak{S}_2, R_2)$ be two IFSHG.

An isomorphism between two IFSHG, is denoted by $\tilde{H}_1 \cong \tilde{H}_2$, is a bijective mapping $\mathfrak{I} : X \rightarrow X'$ which satisfies the following conditions,

- (i) $\mu_{1i}v_i(a_i) = \mu_{2i} [\mathfrak{I}(v_i)](a_i)$ and $v_{1i}v_i(a_i) = v_{2i} [\mathfrak{I}(v_i)](a_i)$, for all $v_i \in \mathfrak{N}.$
- (ii) $\mu_{1ij}(v_i, v_j)(a_i) = \mu_{2ij} [\mathfrak{I}(v_i), \mathfrak{I}(v_j)](a_i)$ and $v_{1ij}(v_i, v_j)(a_i) = v_{2ij} [\mathfrak{I}(v_i), \mathfrak{I}(v_j)](a_i)$, for every $v_i, v_j \in \mathfrak{S}.$

Definition 3.13. Let $\tilde{H}_1 = (\mathfrak{N}_1, \mathfrak{S}_1, R_1)$ and $\tilde{H}_2 = (\mathfrak{N}_2, \mathfrak{S}_2, R_2)$ be two IFSHG.

A homomorphism between two IFSHG, is defined as $\mathfrak{H} : X \rightarrow X'$, is a mapping which satisfies the following conditions,

- (i) $\mu_{1i}v_i(a_i) \geq \mu_{2i} [\mathfrak{H}(v_i)](a_i)$ and $v_{1i}v_i(a_i) \leq v_{2i} [\mathfrak{H}(v_i)](a_i)$, for all $v_i \in \mathfrak{N}.$
- (ii) $\mu_{1ij}(v_i, v_j)(a_i) \geq \mu_{2ij} [\mathfrak{H}(v_i), \mathfrak{H}(v_j)](a_i)$ and $v_{1ij}(v_i, v_j)(a_i) \leq v_{2ij} [\mathfrak{H}(v_i), \mathfrak{H}(v_j)](a_i)$, for every $v_i, v_j \in \mathfrak{S}.$

Definition 3.14. Let $\tilde{H}_1 = (\mathfrak{N}_1, \mathfrak{S}_1, R_1)$ and $\tilde{H}_2 = (\mathfrak{N}_2, \mathfrak{S}_2, R_2)$ be two IFSHG.

A weak isomorphism between two IFSHG, is defined as $\mathfrak{I} : X \rightarrow X'$, is a bijective homomorphism which satisfies the following conditions,

- (i) $\mu_{1i}v_i(a_i) = \mu_{2i} [\mathfrak{I}(v_i)](a_i)$ and $v_{1i}v_i(a_i) = v_{2i} [\mathfrak{I}(v_i)](a_i)$, for all $v_i \in \mathfrak{N}.$

Definition 3.15. Let $\tilde{H}_1 = (\mathfrak{N}_1, \mathfrak{S}_1, R_1)$ and $\tilde{H}_2 = (\mathfrak{N}_2, \mathfrak{S}_2, R_2)$ be two IFSHG.

A co-weak isomorphism between two IFSHG, is defined as $\mathfrak{I} : X \rightarrow X'$, is a bijective homomorphism which satisfies the following conditions,

- (i) $\mu_{1ij}(v_i, v_j)(a_i) = \mu_{2ij} [\mathfrak{I}(v_i), \mathfrak{I}(v_j)](a_i)$ and $v_{1ij}(v_i, v_j)(a_i) = v_{2ij} [\mathfrak{I}(v_i), \mathfrak{I}(v_j)](a_i)$, for every $v_i, v_j \in \mathfrak{S}.$

Definition 3.16. Let $\tilde{H}_1 = (\mathfrak{N}_1, \mathfrak{S}_1, R_1)$ and $\tilde{H}_2 = (\mathfrak{N}_2, \mathfrak{S}_2, R_2)$ be two IFSHG. Then bijective function $\mathfrak{I}: X \rightarrow X'$ is said to be a soft morphism or soft \mathfrak{R} -morphism, if there exists a positive numbers $\mathfrak{R}_1, \mathfrak{R}_2$, such that

- (i) $\mu_{2i} [T(v_i)](a_i) = \mathfrak{R}_1 [\mu_{1i} \langle \mathfrak{I}(v_i)(a_i) \rangle]$ and $v_{2i} [T(v_i)](a_i) = \mathfrak{R}_1 [v_{1i} \langle \mathfrak{I}(v_i)(a_i) \rangle]$, for all $v_i \in \mathfrak{N}$.
- (ii) $\mu_{2ij} [T(v_i), T(v_j)](a_i) = \mathfrak{R}_2 [\mu_{1ij} \langle \mathfrak{I}(v_i v_j)(a_i) \rangle]$ and $v_{2ij} [T(v_i), T(v_j)](a_i) = \mathfrak{R}_2 [v_{1ij} \langle \mathfrak{I}(v_i v_j)(a_i) \rangle]$, for every $v_i, v_j \in \mathfrak{S}$.

Theorem 1. The degrees of vertices of isomorphic intuitionistic fuzzy soft hypergraphs may or may not be preserved.

Proof. The proof is obvious and it is exhibited in Example 6.

Example 6. Let $\tilde{H}_1 = (\mathfrak{N}_1, \mathfrak{S}_1, R_1)$ and $\tilde{H}_2 = (\mathfrak{N}_2, \mathfrak{S}_2, R_2)$ be two IFSHG.

Consider a mapping $\mathfrak{I}: X \rightarrow X'$ by $T(u_1) = u_1, \mathfrak{I}(u_2) = u_2, \mathfrak{I}(u_3) = u_3, \mathfrak{I}(u_4) = u_4, \mathfrak{I}(u_5) = u_5$.

Clearly $deg(u_4)$ and $\mathfrak{I}(u_4) = u_4$ is $(0.5, 0.1)$ under parameterization and $deg(u_2)$ and $\mathfrak{I}(u_2) = u_2$ is $(0.7, 0.2)$ under parameterization. Similarly, the degree of all the vertices may or may not be preserved. But for the parameters a_1 and a_2 ,

$\mathfrak{S}_1(u_1, u_2)(a_i) \neq \mathfrak{S}_1(\mathfrak{I}(u_1), \mathfrak{I}(u_2))(a_i)$. That is, the hyperedges do not remain invariant showing that \tilde{H}_1 and \tilde{H}_2 are not isomorphic.

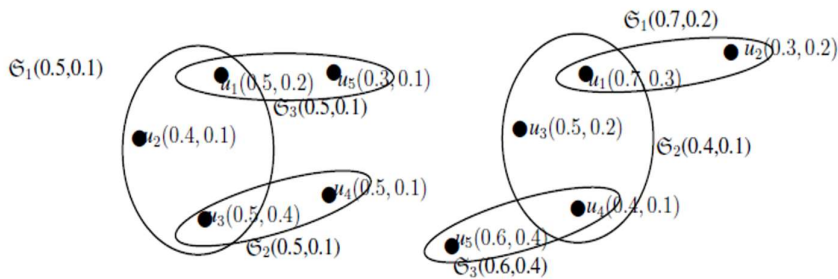


Figure 3: The IFSHG $\tilde{H} = \{ \tilde{H}(a_1), \tilde{H}(a_2) \}$ corresponding to the parameters (a_1, a_2)

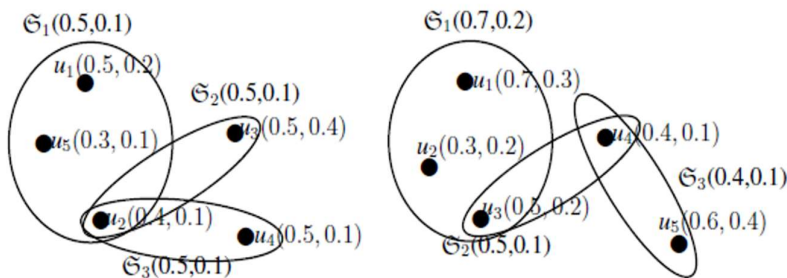


Figure 4: The IFSHG $\tilde{H} = \{ \tilde{H}(a_1), \tilde{H}(a_2) \}$ corresponding to the parameters (a_1, a_2)

Remark 3. In IFSHG, the following conditions are true.

1. The order of weak isomorphism are same.
2. The size of Co-weak isomorphism are same.
3. If the size and order of two IFSHG are equal then it might not be isomorphic.
4. If the degrees of corresponding vertices of IFSHG under parameterization remains invariant then the IFSHG may not be isomorphic.

Theorem 2. *Isomorphism between IFSHG is an equivalence relation.*

Proof. Let $\tilde{H}_1 = (\mathfrak{N}_1, \mathfrak{S}_1, R_1)$, $\tilde{H}_2 = (\mathfrak{N}_2, \mathfrak{S}_2, R)$ and $\tilde{H}_3 = (\mathfrak{N}_3, \mathfrak{S}_3, R)$ be IFSHG with the underlying sets X, X', X'' respectively, and R be the set of parameters.

(i) Reflexive

Consider the identity mapping $\mathfrak{T} : X \rightarrow X$ such that $\mathfrak{T}(u_i)(a_i) = u_i(a_i)$ for all $(u_i) \in X$, Then \mathfrak{T} is a bijective satisfying the following conditions.

$$\mu_{1i}(u_i, v_i)(a_i) = \mu_{1i}[\mathfrak{T}(u_i), \mathfrak{T}(v_i)](a_i) \text{ for all } u_i, v_i \in X \text{ and } a_i \in R.$$

$$v_{1i}(u_i, v_i)(a_i) = v_{1i}[\mathfrak{T}(u_i), \mathfrak{T}(v_i)](a_i)$$

Hence \mathfrak{T} is an isomorphism of IFSHG to itself.

Therefore, it satisfies the reflexive relation.

(ii) Symmetric

Let $\mathfrak{T} : X \rightarrow X'$ be an isomorphism of \tilde{H}_1 onto \tilde{H}_2 , the \mathfrak{T} is a bijective satisfying $\mathfrak{T}(u_i)(a_i) = u_{2i}(a_i)$ for all $u_{2i} \in X'$

$$\mu_{1i}(u_i, v_i)(a_i) = \mu_{2i}[\mathfrak{T}(u_i), \mathfrak{T}(v_i)](a_i) \text{ for all } u_i, v_i \in X \text{ and } a_i \in R.$$

$$v_{1i}(u_i, v_i)(a_i) = v_{2i}[\mathfrak{T}(u_i), \mathfrak{T}(v_i)](a_i)$$

As \mathfrak{T} is a bijective $\mathfrak{T}^{-1}(u_{2i})(a_i) = (u_i)(a_i)$ for every $u_{2i} \in X'$

$$\mu_{1i}[\mathfrak{T}^{-1}(u_{2i}), \mathfrak{T}^{-1}(v_{2i})](a_i) = \mu_{1i}(u_{2i}, v_{2i})(a_i), \text{ for all } u_{2i}, v_{2i} \in X' \text{ and } a_i \in R.$$

Hence we get a one to one onto mapping $\mathfrak{T}^{-1} : X' \rightarrow X$ which is an isomorphism from \tilde{H}_2 to \tilde{H}_1 .

$$\text{ie) } \tilde{H}_1 \cong \tilde{H}_2 \Rightarrow \tilde{H}_2 \cong \tilde{H}_1$$

(iii) Transitive

Let $\mathfrak{T} : X \rightarrow X'$ and $\mathfrak{T}' : X' \rightarrow X''$ be an isomorphisms of IFSHG \tilde{H}_1 onto \tilde{H}_2 and \tilde{H}_2 onto \tilde{H}_3 respectively, then

$$(\mathfrak{T}' \circ \mathfrak{T})(u_i)(a_i) = \mathfrak{T}'[\mathfrak{T}(u_i)](a_i) \text{ for all } u_i \in X$$

As $\mathfrak{T} : X \rightarrow X'$ is an isomorphism,

$$\mathfrak{T}(u_i)(a_i) = u_{2i}(a_i) \text{ for all } (u_{2i}) \in X'$$

$$\mu_{1i}(u_i, v_i)(a_i) = \mu_{2i}[\mathfrak{T}(u_i), \mathfrak{T}(v_i)](a_i) \text{ for all } u_i, v_i \in X \text{ and } a_i \in R.$$

$$v_{1i}(u_i, v_i)(a_i) = v_{2i}[\mathfrak{I}(u_i), \mathfrak{I}(v_i)](a_i)$$

ie $\mu_{1i}(u_i, v_i)(a_i) = \mu_{2i}(u_i, v_i)(a_i)$

$$v_{1i}(u_i, v_i)(a_i) = v_{2i}(u_i, v_i)(a_i)$$

As $\mathfrak{I}': X' \rightarrow X''$ is an isomorphism from X' to X'' , then we have

$$\mathfrak{I}': (u_j)(a_i) = u_{3i}(a_i) \text{ for all } u_{3i} \in X''$$

$$\mu_{2i}(u_j, v_j)(a_i) = \mu_{3i}(\mathfrak{I}'(u_j), \mathfrak{I}'(v_j))(a_i)$$

$$v_{2i}(u_j, v_j)(a_i) = v_{3i}(\mathfrak{I}'(u_j), \mathfrak{I}'(v_j))(a_i)$$

using above, We get

$$\mu_{1i}(u_i, v_i)(a_i) = \mu_{2i}(u_i, v_i)(a_i) = \mu_{3i}(\mathfrak{I}'(u_i), \mathfrak{I}'(v_i))(a_i)$$

$$= \mu_{3i}[(\mathfrak{I}'(\mathfrak{I}(u_i))), \mathfrak{I}'(\mathfrak{I}(v_i))]$$

$$v_{1i}(u_i, v_i)(a_i) = v_{2i}(u_i, v_i)(a_i) = v_{3i}(\mathfrak{I}'(u_i), \mathfrak{I}'(v_i))(a_i)$$

$$= v_{3i}[(\mathfrak{I}'(\mathfrak{I}(u_i))), \mathfrak{I}'(\mathfrak{I}(v_i))]$$

Therefore $\mathfrak{I}' \circ \mathfrak{I}$ is an isomorphism between X and X'' .

Hence isomorphism between Intuitionistic fuzzy soft hypergraphs is an equivalence relation.

Theorem 3. *Weak isomorphism between IFSHG satisfies the partial order relation.*

Proof. It can be easily proved by the same arguments given in the above theorem.

Theorem 4. *Let $\tilde{H}_1 = (\mathfrak{N}_1, \mathfrak{S}_1, R_1)$, $\tilde{H}_2 = (\mathfrak{N}_2, \mathfrak{S}_2, R)$ be two IFSHG. If \tilde{H}_1 is co-weak isomorphic to \tilde{H}_2 and \tilde{H}_1 is regular, then \tilde{H}_2 is regular too under parameterization.*

Proof. Let IFSHG $\tilde{H}_1 = (\mathfrak{N}_1, \mathfrak{S}_1, R_1)$ is a co-weak isomorphic to \tilde{H}_2 , then there exists a co-weak isomorphism $\mathfrak{I} : X \rightarrow X'$, which is a bijective homomorphism that satisfies the following conditions,

(i) $\mu_{1ij}(v_i, v_j)(a_i) = \mu_{2ij}[\mathfrak{I}(v_i), \mathfrak{I}(v_j)](a_i)$ and

$$v_{1ij}(v_i, v_j)(a_i) = v_{2ij}[\mathfrak{I}(v_i), \mathfrak{I}(v_j)](a_i), \text{ for every } v_i, v_j \in \mathfrak{S}.$$

Since \tilde{H}_1 is regular,

$$\sum_{v_i \neq v_j, v_i, v_j \in \mathfrak{N}_1} \mu_{2ij}(v_i v_j)(a_i) = \text{constant},$$

$$\sum_{v_i \neq v_j, v_i, v_j \in \mathfrak{N}_1} v_{2ij}(v_i v_j)(a_i) = \text{constant}$$

Now,

$$\sum_{\mathfrak{I}(v_i) \neq \mathfrak{I}(v_j)} \mu_{2ij}[\mathfrak{I}(v_i), \mathfrak{I}(v_j)](a_i) = \sum_{v_i \neq v_j, v_i, v_j \in \mathfrak{N}_1} \mu_{1ij}(v_i v_j)(a_i) = \text{constant},$$

$$\sum_{\mathfrak{I}(v_i) \neq \mathfrak{I}(v_j)} v_{2ij}[\mathfrak{I}(v_i), \mathfrak{I}(v_j)](a_i) = \sum_{v_i \neq v_j, v_i, v_j \in \mathfrak{N}_1} v_{1ij}(v_i v_j)(a_i) = \text{constant}$$

Remark 4. Let $\tilde{H}_1 = (\mathfrak{N}_1, \mathfrak{S}_1, R_1)$, $\tilde{H}_2 = (\mathfrak{N}_2, \mathfrak{S}_2, R)$ be two IFSHGs. If \tilde{H}_1 is weak isomorphic to \tilde{H}_2 and \tilde{H}_1 is regular, then \tilde{H}_2 need not to be regular.

Theorem 5. Let $\tilde{H} = (\mathfrak{R}, \mathfrak{S})$ be an IFSHG. Then $(\mathfrak{S}_\mu : \mathfrak{R} \rightarrow [0, 1], \mathfrak{S}_\nu : \mathfrak{R} \rightarrow [0, 1])$ is a constant function iff the following conditions are equivalent.

- (i) \tilde{H} is a regular IFSHG.
- (ii) \tilde{H} is totally regular IFSHG.

Proof. Suppose that $(\mathfrak{S}_\mu, \mathfrak{S}_\nu)$ be a constant function.

Let $\mathfrak{S}_\mu(v_1) = C_1$ and $\mathfrak{S}_\nu(v_1) = C_2$ for the parameter $a_1 \in R$ and $v_1 \in \mathfrak{S}$.

(i) \Rightarrow (ii) Assume that \tilde{H} is a (k_i, k'_i) -regular IFSHG.

Let $deg_\mu(v_1(a_1)) = k_1$ and $deg_\nu(v_1(a_1)) = k'_1$.

Then we have, $deg_\mu[v_1(a_1)] = deg_\mu(v_1(a_1)) + \mathfrak{S}_\mu(v_1(a_1))$ and

$deg_\nu[v_1(a_1)] = deg_\nu(v_1(a_1)) + \mathfrak{S}_\nu(v_1(a_1))$

Thus $deg_\mu[v_1(a_1)] = k_1 + C_1$ and $deg_\nu[v_1(a_1)] = k'_1 + C_2$.

Hence \tilde{H} is totally regular IFSHG.

(ii) \Rightarrow (i) Assume that \tilde{H} is a (p_i, p'_i) -totally regular IFSHG.

Let $deg_\mu[v_1(a_1)] = p_1$ and $deg_\nu[v_1(a_1)] = p'_1$.

Then we have $deg_\mu[v_1(a_1)] = deg_\mu(v_1(a_1)) + \mathfrak{S}_\mu(v_1(a_1))$ and

$deg_\nu[v_1(a_1)] = deg_\nu(v_1(a_1)) + \mathfrak{S}_\nu(v_1(a_1))$

$\Rightarrow deg_\mu(v_1(a_1)) + \mathfrak{S}_\mu(v_1(a_1)) = p_1, deg_\nu(v_1(a_1)) + \mathfrak{S}_\nu(v_1(a_1)) = p'_1$

$\Rightarrow deg_\mu(v_1(a_1)) + C_1 = p_1, deg_\nu(v_1(a_1)) + C_2 = p'_1$

$\Rightarrow deg_\mu(v_1(a_1)) = p_1 - C_1, deg_\nu(v_1(a_1)) = p'_1 - C_2$, for $a_1 \in R$ and $v_1 \in \mathfrak{S}$

Thus \tilde{H} is a regular IFSHG.

Hence (i) and (ii) are equivalent.

Conversely, Assume that (i) and (ii) are equivalent.

That is \tilde{H} is a regular IFSHG iff \tilde{H} is a totally regular IFSHG.

Suppose that $(\mathfrak{S}_\mu, \mathfrak{S}_\nu)$ is not a constant function and $\mathfrak{S}_i(v_1)$ and $\mathfrak{S}_i(v_2)$ is not equal for some $v_1, v_2 \in \mathfrak{R}$ corresponding to the parameter a_1 . If \tilde{H} is a (k_i, k'_i) -regular IFSHG, then $deg(v_1(a_1)) = (k_1, k'_1)$ - for all $v_1 \in \mathfrak{S}_i$.

Consider, $deg[v_1(a_1)] = deg(v_1(a_1)) + \mathfrak{S}(v_1(a_1)) = (k_1, k'_1) + \mathfrak{S}(v_1(a_1))$ and

$deg[v_2(a_1)] = deg(v_2(a_1)) + \mathfrak{S}(v_2(a_1)) = (k_2, k'_2) + \mathfrak{S}(v_2(a_1))$.

Then $\mathfrak{S}_i(v_1)$ and $\mathfrak{S}_i(v_2)$ is not equal for some $v_1, v_2 \in \mathfrak{R}$ corresponding to the parameter a_1 .

Thus $deg[v_1(a_1)]$ and $deg[v_2(a_1)]$ are not equal. Hence \tilde{H} is not a totally regular IFSHG, which is a contradiction.

Let \tilde{H} is totally regular IFSHG. Then $deg[v_1(a_1)] = deg[v_2(a_1)]$.

That is $deg(v_1(a_1)) + \mathfrak{S}(v_1(a_1)) = deg(v_2(a_1)) + \mathfrak{S}(v_2(a_1))$ and

$deg(v_1(a_1)) - deg(v_2(a_1)) = \mathfrak{S}(v_2(a_1)) - \mathfrak{S}(v_1(a_1))$.

Since $deg(v_1(a_1)) - deg(v_2(a_1)) \neq 0$ and $\mathfrak{S}(v_2(a_1)) - \mathfrak{S}(v_1(a_1)) \neq 0$.

Thus $deg[v_1(a_1)] \neq deg[v_2(a_1)]$. So \tilde{H} is not a regular IFSHG, which is a contradiction to our assumption,

Hence $(\mathfrak{S}_\mu, \mathfrak{S}_\nu)$ must be a constant function.

Theorem 6. Let $\tilde{H} = (\mathfrak{R}, \mathfrak{S})$ be an IFSHG. If \tilde{H} is both regular and totally

regular, then $(\mathfrak{S}_\mu, \mathfrak{S}_\nu)$ is a constant function.

Proof. Let $\tilde{H} = (\mathfrak{R}, \mathfrak{S})$ be an IFSHG and it is both regular and totally regular.

Let $deg_\mu[v_1(a_i)] = p_1$ and $deg_\nu[v_1(a_i)] = p'_1$, for all $a_i \in R$ and $v_i \in \mathfrak{S}$.

Let $deg_\mu(v_i(a_i)) = p_n$ and $deg_\nu(v_i(a_i)) = p'_n$, for all $a_i \in R$ and $v_i \in \mathfrak{S}$.

Consider, $deg_\mu[v_i(a_i)] = p_1$, for all $v_i \in \mathfrak{S}$

$$\Leftrightarrow deg_\mu(v_i(a_i)) + \mathfrak{S}_\mu(v_i(a_i)) = p_1$$

$$\Leftrightarrow p_n + \mathfrak{S}_\mu(v_i(a_i)) = p_1$$

$$\Leftrightarrow \mathfrak{S}_\mu(v_i(a_i)) = p_1 - p_n, \text{ for all } v_i \in \mathfrak{S} \text{ and } a_i \in R.$$

Consider, $deg_\nu(v_i(a_i)) = p'_1$, for all $v_i \in \mathfrak{S}$

$$\Leftrightarrow deg_\nu(v_i(a_i)) + \mathfrak{S}_\nu(v_i(a_i)) = p'_1$$

$$\Leftrightarrow p_n + \mathfrak{S}_\nu(v_i(a_i)) = p'_1$$

$$\Leftrightarrow \mathfrak{S}_\nu(v_i(a_i)) = p'_1 - p_n, \text{ for all } v_i \in \mathfrak{S} \text{ and } a_i \in R.$$

Hence $(\mathfrak{S}_\mu, \mathfrak{S}_\nu)$ is a constant function.

Note: The converse of the theorem need not be true.

3 Conclusion

Hypergraphs are thought to be the most efficient representation for dealing with complex practical problems in real life. An IFSS is a fuzzy soft set extension that is used to deal with uncertain information under complexity based on the parameters. So, by incorporating IFSS and hypergraphs, a concept of IFSHGs is provided, as well as discussion of regular - IFSHGs and totally regular IFSHGs. It is revealed that isomorphism between IFHG is an equivalence relation and that weak isomorphism satisfies the partial order relation. Soft K -morphism is also introduced. Furthermore, the author intends to expand his research into soft K- morphism IFSHGs and their properties.

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