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FUZZY NUMBER GRACEFUL LABELING GRAPHS

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ABSTRACT: Certain idea about the fuzzy number graceful labeling graph of a fuzzy graph is given and some related concepts of the fuzzy number graceful labeling graph are also discussed.
KEY WORDS: Fuzzy set, fuzzy graph, fuzzy number graceful labeling.

INTRODUCTION

First, fuzzy set had been introduced by Zadeh [16]. Succeeding years, fuzzy set was grown in different ways. The following are extension of fuzzy set, they are vague set, intuitionistic fuzzy set, bipolar valued fuzzy set and etc. In 1975, fuzzy graph was introduced by Rosenfeld [10], with modification of fuzzy graph was introduced by Arjunan.K and C.Subramani[2] and it was extended to many area. In similar way, [3], [4], [5], [6], [7], [8], [9], [11], [12], [13], [14] and [15] were useful to write this paper. K.Arjunan et all.[1] have given an idea about the fuzzy number graceful labeling graph. In this paper, graceful labeling is generalized, particularly fuzzy number graceful labeling, the triangular fuzzy number is used in this paper but we have different types of fuzzy number.

1.PRELIMINARIES

Definition 1.1. [16] A fuzzy set $\tilde{N}$ on the given universal set Q is a set of ordered pairs $\tilde{N} = \{(x, F_m(x)): x \in Q \}$, where $F_m: Q \rightarrow [0,1]$, is called membership function.

Definition 1.2. [16] The k − cut of a fuzzy set $\tilde{N}$, is defined by $\tilde{N}L = \{z: F_m(z) \geq k\}$, where $k \in [0, 1]$.

Definition 1.3. [16] Let Φ, Θ be two fuzzy subsets of a set G. The following relations and operations are defined as:

(i) $\Phi \subset \Theta$ means $\Phi(t) \leq \Theta(t)$, for all $t \in G$.

(ii) $\Phi = \Theta$ means $\Phi(t) = \Theta(t)$, for all $t \in G$.

(iii) $\Phi^c(t) = 1 - \Phi(t)$, for all $t \in G$.

(iv) $\Phi \cap \Theta = \{\langle t, \min(\Phi(t), \Theta(t)) \rangle / t \in G\}$.

(v) $\Phi \cup \Theta = \{\langle t, \max(\Phi(t), \Theta(t)) \rangle / t \in G\}$.

Definition 1.4. [14] A fuzzy subset of the real line R, whose membership function F_m satisfies the following situation, is a generalized fuzzy number $\tilde{N}$

(i) F_m is a continuous mapping from R to the closed interval [0, 1],

(ii) $F_m = 0$, $-\infty < x \leq a_1$,

(iii) $F_m = L(x)$ is strictly increasing on $[a_1, a_2]$,

(iv) $F_m = W_N$, $a_2 \leq x \leq a_3$,

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(v) $F_m = R(x)$ is strictly decreasing on $[a_3, a_4]$,

(vi) $F_m = 0$, $a_4 \le x \le \infty$,

where $0 < W_N \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Moreover, this kind of generalized fuzzy number is denoted as $\tilde{N} = (a_1, a_2, a_3, a_4; W_N)_{LR}$ when $W_N = 1$, it can be simplified as $\tilde{N} = (a_1, a_2, a_3, a_4)_{LR}$.

Definition 1.5. [14] The fuzzy set $\tilde{N} = (a_1, a_2, a_3)$, where $a_1 < a_2 < a_3$ and described on R, is called a triangular fuzzy number, the membership function of $\tilde{N}$ is given by

$$F_m = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, a_2 \le x \le a_3 \\ 0, otherwise \end{cases}$$

Definition 1.6. The collection S = { $\tilde{0}, \tilde{1}, \tilde{2}, \ldots$ } of triangular fuzzy numbers is called non-negative triangular fuzzy numbers if $\tilde{a} = (a_1, a, a_3)$, $\tilde{a} \in S$, $a_1, a_3 \ge 0$. For example $\tilde{1} = (0,1,2)$ and $\tilde{0} = (0,0,0)$.

Definition 1.7. [11] The function principle was introduced by Shan-Huo Chen [10] to treat fuzzy arithmetical operations. This principle is used for the process for addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{N} = (a_1, a_2, a_3)$ and $\hat{U} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) the addition of $\tilde{N}$ and $\hat{U}$ is $\tilde{N} + \hat{U} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$, where $a_1, a_2, a_3, b_1, b_2, b_3$ are any real numbers.

(ii) The multiplication of $\tilde{N}$ and $\hat{U}$ is $\tilde{N} \times \hat{U} = (a_1b_1, a_2b_2, a_3b_3)$, where $a_1, a_2, a_3, b_1, b_2, b_3$ are all non zero positive real numbers.

(iii) $-\hat{U} = (-b_3, -b_2, -b_1)$, the subtraction of $\hat{U}$ from $\tilde{N}$ is $\tilde{N} - \hat{U} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$, where $a_1, a_2, a_3, b_1, b_2, b_3$ are any real numbers.

(iv) $\frac{1}{\hat{U}} = \hat{U}^{-1} = (\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1})$, where b_1, b_2, b_3 are all non zero positive real numbers, then the division of $\tilde{N}$ and $\hat{U}$ is $\frac{\hat{N}}{\hat{U}} = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})$.

(v) For any real number K,

$K\tilde{N} = (Ka_1, Ka_2, Ka_3)$ if $K > 0$

$K\tilde{N} = (Ka_3, Ka_2, Ka_1)$ if $K < 0$.

Definition 1.8.[14] (i). Defuzzification of $\tilde{N} = (a_1, a_2, a_3)$ can be done by Graded Mean Integration Representation Method. It is defined as

$$G(\hat{N}) = \frac{1}{2}\frac{\int_0^1 l[a_1 + l(a_2 - a_1) + a_3 - l(a_3 - a_2)]dl}{\int_0^1 l dl} = \frac{a_1 + 4a_2 + a_3}{6}.$$

(ii). [3] The signed distance of $\tilde{N}$ is defined as

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$$d(\hat{N},0)=S(\hat{N})=\frac{1}{2}\int_{0}^{1}a_1+l(a_2-a_1)+a_3-l(a_3-a_2)]dl=\frac{a_1+2a_2+a_3}{4}.$$

(iii). Total Integral Value Method (TI) of Ñ is defined as

$$TI(\hat{N})=\int_{0}^{1}a_1+l(a_2-a_1)+a_3-l(a_3-a_2)]dl=\frac{a_1+2a_2+a_3}{2}.$$

Definition 1.9. [2] Let Φ be a FS in a set G. The strongest fuzzy relation Ψ is an ordered pair Ψ = { ⟨ (c_1, d_1), $\Psi(c_1, d_1)$ ⟩ / $c_1, d_1 \in G$ } on G which is a fuzzy relation on Φ and is defined by $\Psi(c_1, d_1) = \min\{\Phi(c_1), \Phi(d_1)\}$ for all $c_1, d_1 \in G$.

Definition 1.10. [2] Let S be any nonempty set and L be any set with a function ℑ: L →S×S. Then Φ is a $_F$S of S, R is a fuzzy relation on S with respect to Φ and Θ is a $_F$S of L such that $\Theta(e) \leq \underset{e\in\Im^{-1}(x,y)}{R}(x,y)$. Then the ordered triple Ω = (Φ, Θ, ℑ) is called a fuzzy graph ($_F$G), here the elements of Φ are fuzzy points or fuzzy vertices ($_F$V) and the elements of Θ are fuzzy lines or fuzzy edges ($_F$E) of the $_F$G Ω. Let ℑ(a) = (c_1, d_1). Then $(c_1, \Phi(c_1))$, $(d_1, \Phi(d_1))$ are adjacent $_F$Vs and $(c_1, \Phi(c_1))$, $_F$E (a, Θ(a)) are incident with each other. Let two distinct $_F$Es $(a_1, \Theta(a_1))$ and $(a_2, \Theta(a_2))$ are incident with a common $_F$V. They are called adjacent $_F$Es. Let Φ^* = { w∈S / Φ(w) > 0 } and Θ^* = { c∈L / Θ(c) > 0. Then $\Omega^* = (\Phi^*, \Theta^*, \Im)$ is a crisp graph.

Definition 1.11. [2] *Let* Ω = (Φ, Θ, ℑ) *be a* $_F$G. A $_F$E is called a fuzzy loop (FL) if its end $_F$Vs of the $_F$E are joined with same $_F$V.

Definition 1.12. [2] *Let* Ω = (Φ, Θ, ℑ) *be a* $_F$G. Then the $_F$G Ω is called a fuzzy simple graph($_F$SG) if it has neither fuzzy multiple lines nor $_F$Ls.

Example 1.13. Consider the $_F$G Ω = (Φ, Θ, ℑ), here S = { a_1, b_1, c_1, d_1, e_1}, L = {x, y, z, u, v, w, p } and ℑ: L →S×S is defined by ℑ(x) = (a_1, b_1), ℑ(y) = (b_1, b_1), ℑ(z) = (b_1, c_1), ℑ(u) = (c_1, d_1), ℑ(v) = (c_1, d_1), ℑ(w) = (d_1, e_1), ℑ(p) = (a_1, e_1). A $_F$S Φ = { $(a_1, 0.5)$, $(b_1, 0.4)$, $(c_1, 0.6)$, $(d_1, 0.7)$, $(e_1, 0.3)$ } of S. A fuzzy relation R = { $((a_1, a_1), 0.5)$, $((a_1, b_1), 0.4)$, $((a_1, c_1), 0.5)$, $((a_1, d_1), 0.5)$, $((a_1, e_1), 0.3)$, $((b_1, a_1), 0.4)$, $((b_1, b_1), 0.4)$, $((b_1, c_1), 0.4)$, $((b_1, d_1), 0.4)$, $((b_1, e_1), 0.3)$, $((c_1, a_1), 0.5)$, $((c_1, b_1), 0.4)$, $((c_1, c_1), 0.6)$, $((c_1, d_1), 0.6)$, $((c_1, e_1), 0.3)$, $((d_1, a_1), 0.5)$, $((d_1, b_1), 0.4)$, $((d_1, c_1), 0.5)$, $((d_1, d_1), 0.7)$, $((d_1, e_1), 0.3)$, $((e_1, a_1), 0.3)$, $((e_1, b_1), 0.3)$, $((e_1, c_1), 0.3)$, $((e_1, d_1), 0.3)$, $((e_1, e_1), 0.3)$} on S with respect to Φ and a $_F$S Θ = { (x, 0.3), (y, 0.2), (z, 0.1), (u, 0.4), (v, 0.5), (w, 0.15), (p, 0.25)} of L.

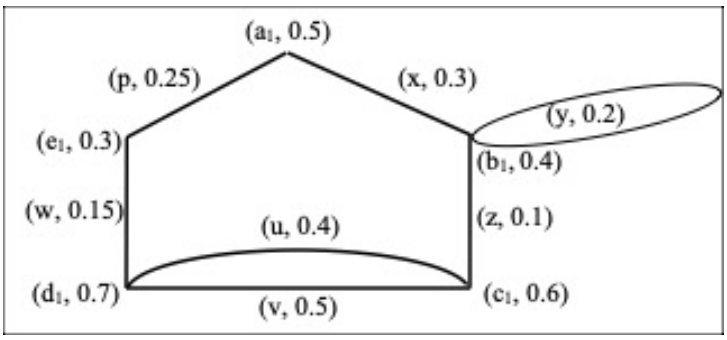

Fig. 1.1.

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In figure 1.1, (i) (a_1, 0.5), (b_1, 0.4), (c_1, 0.6), (d_1, 0.7) and (e_1, 0.3) are $_F$Vs.
(ii) (x, 0.3), (y, 0.2), (z, 0.1), (u, 0.4), (v, 0.5), (w, 0.15) and (p, 0.25) are $_F$Es.
(iii) (a_1, 0.5) (b_1, 0.4), (b_1, 0.4) (c_1, 0.6), (c_1, 0.6) (d_1, 0.7), (d_1, 0.7) (e_1, 0.3) and (e_1, 0.3) (a_1, 0.5) are adjacent $_F$Vs.
(iv) For example, (x, 0.3) join with (a_1, 0.5) and (b_1, 0.4), so it is incident with (a_1, 0.5) and (b_1, 0.4).
(v) (x, 0.3) and (p, 0.25) are adjacent $_F$Es.
(vi) (y, 0.2) is a $_F$L.
(vii) (u, 0.4) and (v, 0.5) are fuzzy multiple edges.
(viii) The given $_F$G is not a $_F$SG.
(ix) The given $_F$G is a $_F$PG.

Definition 1.14. [2] Let $\Psi = (\rho, \sigma, \Im)$ and $\Omega = (\Phi, \Theta, \Im)$ be two $_F$Gs of G. Then $_F$G Ψ is called a fuzzy subgraph ($_F$S$_u$G) of Ω if $\rho \subseteq \Phi$ and $\sigma \subseteq \Theta$.

Definition 1.15. [2] Let $\Omega = (\Phi, \Theta, \Im)$ be a $_F$G. *Then the degree of a fuzzy vertex* is defined *by* $d(\beta)$ *and* $d(\beta) = \sum_{a \in \Im^{-1}(\alpha,\beta)} \Theta(a) + 2 \sum_{a \in \Im^{-1}(\beta,\beta)} \Theta(a)$.

Defination 1.16. [2] The minimum degree of the $_F$G $\Omega = (\Phi, \Theta, \Im)$ is $\delta(\Omega)$, $\delta(\Omega) = \min\{d(\beta) / \beta \in S\}$ and the maximum degree of Ω is $\Delta(\Omega)$, $\Delta(\Omega) = \max\{d(\beta) / \beta \in S\}$.

Definition 1.17. [2] Let $\Omega = (\Phi, \Theta, \Im)$ be a $_F$G. Then the order of Ω is defined to be $o(\Omega)$, $o(\Omega) = \sum_{\beta \in S} \Phi(\beta)$.

Definition 1.18. [2] Let $\Omega = (\Phi, \Theta, \Im)$ be a $_F$G. Then the size of Ω is defined to be $S(\Omega)$, $S(\Omega) = \sum_{a \in \Im^{-1}(\alpha,\beta)} \Theta(a)$.

Definition 1.19. [5] A $_F$G $\Omega = (\Phi, \Theta, \Im)$ is called fuzzy regular graph if $d(\beta) = r$, for all $\beta \in S$. It is also called a fuzzy $r-$regular graph.

Definition 1.20. [5] A $_F$G $\Omega = (\Phi, \Theta, \Im)$ is called a fuzzy complete graph ($_F$C$_{om}$G) if every pair of distinct $_F$Vs are adjacent and $\Theta(a)_{a \in \Im^{-1}(\alpha,\beta)} = R(\alpha, \beta)$ for all $\alpha, \beta \in S$.

Definition 1.21. [1] A fuzzy path($_F$P$_a$) in a $_F$G $\Omega = (\Phi, \Theta, \Im)$ is an alternating sequence of $_F$Vs and $_F$Es (s_0, $\Phi(s_0)$), (a_1, $\Theta(a_1)$), (s_1, $\Phi(s_1)$), (a_2, $\Theta(a_2)$), (s_2, $\Phi(s_2)$),…,(s_{n-1}, $\Phi(s_{n-1})$), (a_n, $\Theta(a_n)$), (s_n, $\Phi(s_n)$) such that $\Theta(a_i) > 0$, $i = 1, 2, \ldots, n$ and all the $_F$Vs are distinct. Here n is called the length of the $_F$P$_a$. The strength of the $_F$P$_a$ is defined as $\bigcap_{i=1}^{n} \Theta(a_i)$. In other words the strength of $_F$P$_a$ is defined to be the weight of the weakest $_F$E of the $_F$P$_a$. A single $_F$V β may also be considered as a $_F$P$_a$. In this case, the $_F$P$_a$ is of length 0. If a $_F$P$_a$ has length 0, its strength is $\Phi(\beta)$. The consecutive pairs (s_{i-1}, s_i) are called arcs of the $_F$P$_a$.

Definition 1.22. [1] A closed fuzzy path is called a fuzzy cycle ($_F$C$_y$) and the length of the fuzzy cycle is given by the number of fuzzy edges in a fuzzy cycle. A fuzzy cycle of length n is denoted as C_n.

Definition 1.23. Let $\Omega = (\Phi, \Theta, \Im)$ be a fuzzy graph with p fuzzy points and p-1 fuzzy edges

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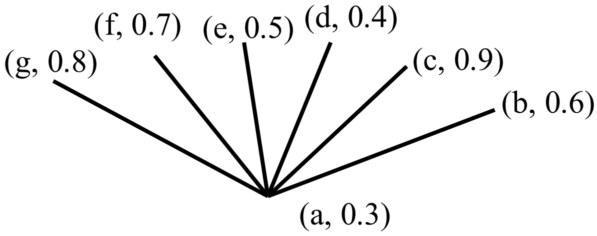
such that exactly one fuzy point is adjacent to every other p-1 fuzzy points. Then Ω is called fuzzy flower graph with p-1 petals.

Example 1.24. A fuzzy flower graph with 6 petals is given below.

Fig. 1.2.



Definition 1.25. A $_F$G $\Omega = (\Phi, \Theta, \Im)$ is fuzzy connected graph ($_F$C$_{on}$G) if any two $_F$Vs are joined by a $_F$P$_a$. Otherwise, Ω is called fuzzy disconnected graph ($_F$DC$_{on}$G).

Definition 1.26. Let $\Omega = (\Phi, \Theta, \Im)$ be a $_F$G. The complement of Ω is defined as $\Omega^c = (\Phi^c, \Theta^c, \Im)$, where $\Phi^c(\alpha) = \Phi(\alpha)$, for all $\alpha \in S$ and $\Theta^c(a) = \min\{\Phi(\alpha), \Phi(\beta)\} - \Theta(a)$ $\forall$ $a \in L$ and $\Im(a) = (\alpha, \beta)$.

Definition 1.27. A $_F$G $\Omega = (\Phi, \Theta, \Im)$ that has no $_F$C$_y$s is called a fuzzy cyclic or a forest. A connected forest is called a fuzzy tree ($_F$T). It is also denoted as f − tree.

2. FUZZY NUMBER LABELING GRAPH

Definition 2.1. [1] A fuzzy number graceful labeling($_{FN}$GfL) of a $_F$G $\Omega = (\Phi, \Theta, \Im)$ having q $_F$Es is an injective mapping $\mathfrak{L} : \Phi(\Omega) \to \{\tilde{0}, \tilde{1}, \tilde{2}, \ldots, \tilde{q}\}$ such that when each $_F$E $\zeta\varsigma$ is assigned the label $|\mathfrak{L}(\zeta) - \mathfrak{L}(\varsigma)|$, the resulting $_F$E labels are distinct. A fuzzy number graceful graph($_{FN}$GfG) is one which admits a $_{FN}$GfL.

Definition 2.2. [1] A fuzzy number nearly graceful labeling($_{FN}$NGfL) of a $_F$G $\Omega = (\Phi, \Theta, \Im)$ having p $_F$Vs and q $_F$Es is an injective mapping $\mathfrak{L} : \Phi(\Omega) \to \{\tilde{0}, \tilde{1}, \tilde{2}, \ldots, \tilde{q} + 1\}$ such that when each $_F$E $\zeta\varsigma$ is assigned the label $|\mathfrak{L}(\zeta) - \mathfrak{L}(\varsigma)|$, the resulting $_F$E labels are distinct. A fuzzy number nearly graceful graph($_{FN}$NGfG) is one which admits a $_{FN}$NGfL.

Theorem 2.3. Every $_F$P$_a$ is a $_{FN}$GfG.

Proof. Let W_m be a $_F$P$_a$ with m $_F$Vs. That is W_m has the number of $_F$Es is m-1. Labeling can begin at either end without loss of generality. The first point at one end is labeled as $\tilde{0}$, the adjacent point is labeled as $\widetilde{m-1}$,the next adjacent, non labeled point is labeled as $\tilde{1}$ and we continue in this manner. Alternate points are incremently increasing by 1 while the remaining points are incremently decreasing by 1. Let s = $[\frac{m}{2}]$. For cases when m is even, the $_F$E labels begining with the leftmost edge in figure are $|(\widetilde{m-1}) - \tilde{0}|$, $|(\widetilde{m-1}) - \tilde{1}|$,..., $|(\widetilde{m-s}) - \widetilde{s-1}|$, all are distinct.

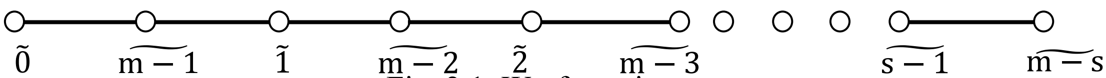


Fig. 2.1. W_m for m is even

For cases when m is odd, the edge labels begining with the leftmost edge in figure are $|(\widetilde{m-1}) - \tilde{0}|$, $|(\widetilde{m-1}) - \tilde{1}|$,..., $|(\widetilde{m-s}) - \tilde{s}|$, all are distinct.

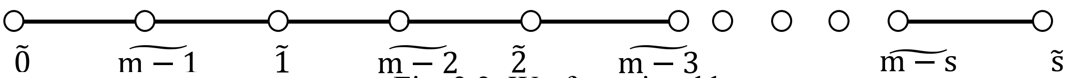


Fig. 2.2. W_m for m is odd

Remark 2.4. Every crisp graceful path is a $_{FN}$Gf path graph.

Example 2.5. A triangular fuzzy number path graph($_{TFN}$P$_a$G) with 6 and 7 $_F$Vs.

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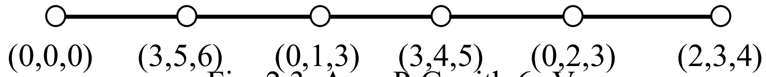


Fig. 2.3. A $_{TFN}P_aG$ with 6 $_F$Vs

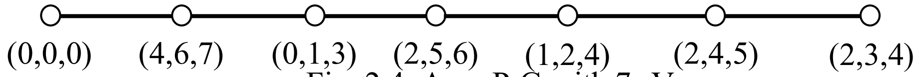


Fig. 2.4. A $_{TFN}P_aG$ with 7 $_F$Vs

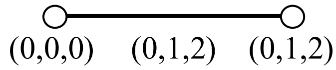
Proposition 2.6. $_F$C$_{om}$G K$_2$ is a $_{FN}$Gf complete graph.

Proof. From the $_F$G, K$_2$ is $_{FN}$Gf complete graph.

Fig. 2.5.

Example 2.7. $_{TFN}$G complete graph K$_2$.

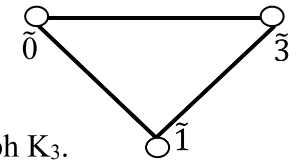
Fig. 2.6.



Proposition 2.8. $_F$C$_{om}$G K$_3$ is a $_{FN}$Gf complete graph.

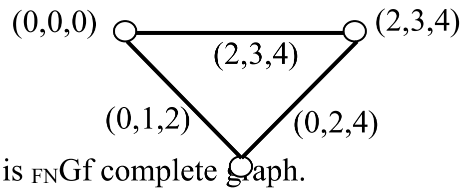
Proof. From the $_F$G, K$_3$ is $_{FN}$Gf complete graph.

Fig. 2.7.



Example 2.9. $_{TFN}$Gf complete graph K$_3$.

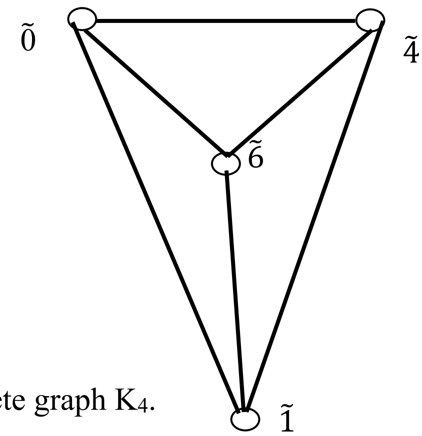
Fig. 2.8.



Proposition 2.10. $_F$C$_{om}$G K$_4$ is $_{FN}$Gf complete graph.

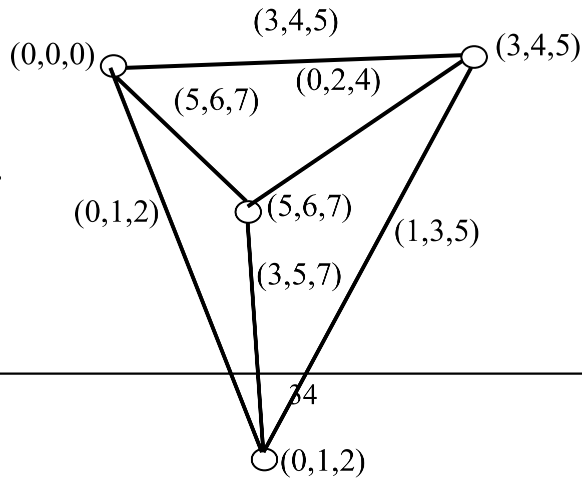
Proof. From the $_F$G, K$_4$ is a $_{FN}$Gf complete graph.

Fig. 2.9.



Example 2.11. $_{TFN}$Gf complete graph K$_4$.

Fig. 2.10.



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Proposition 2.12. ${}_{F}C_{om}G$ K_5 is a ${}_{FN}Gf$ complete graph.

Proof. From the ${}_{F}G$, K_5 is ${}_{FN}Gf$ complete graph.

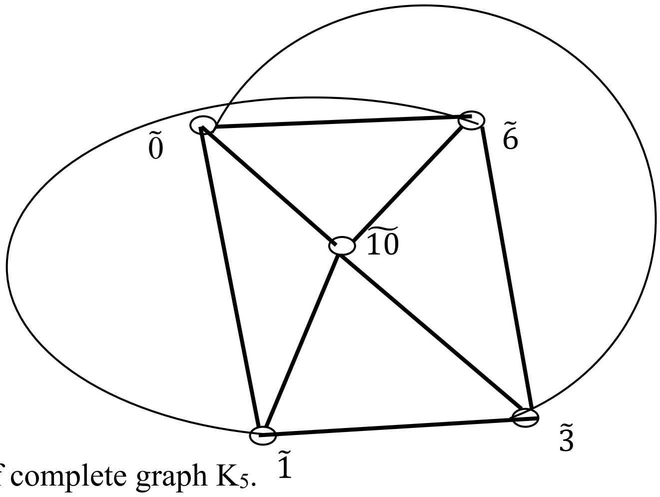

Fig. 2.11.

Example 2.13. ${}_{TFN}Gf$ complete graph K_5.

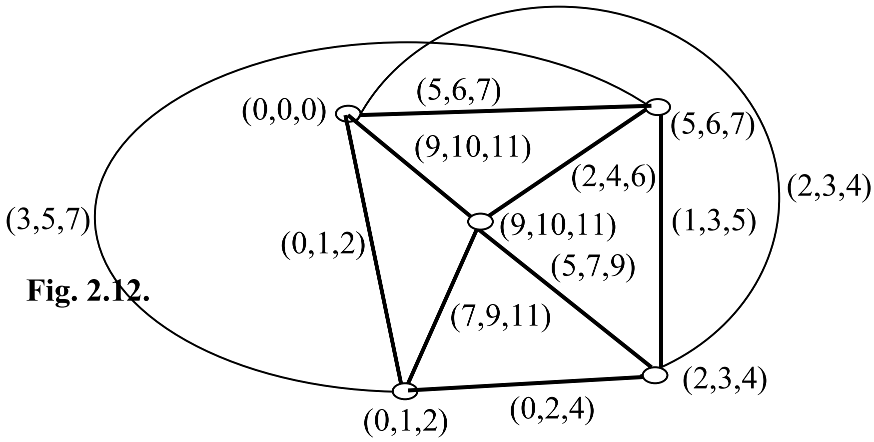

Fig. 2.12.

Remark 2.14. In crisp graph, K_5 is not a graceful complete graph.

Proposition 2.15. ${}_{F}C_{om}G$ K_6 is a ${}_{FN}Gf$ complete graph.

Proof. From the ${}_{F}G$, K_6 is ${}_{FN}Gf$ complete graph.

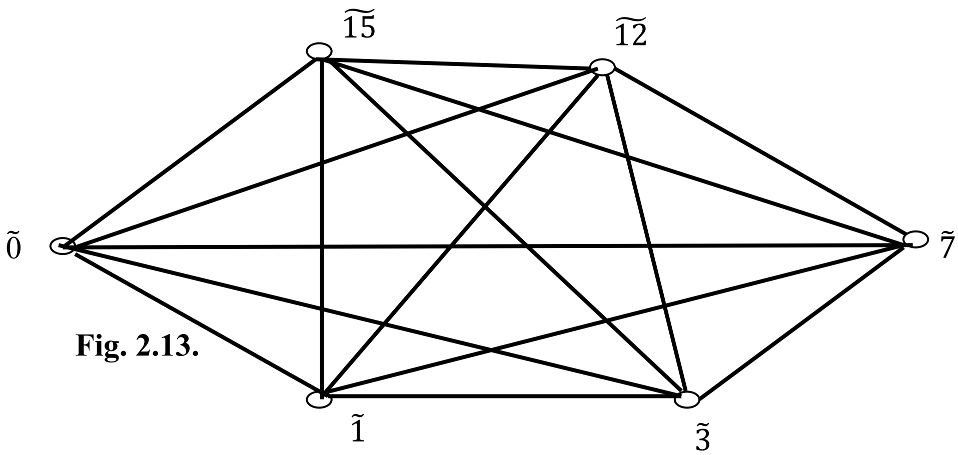

Fig. 2.13.

Example 2.16. ${}_{TFN}Gf$ complete graph K_6.

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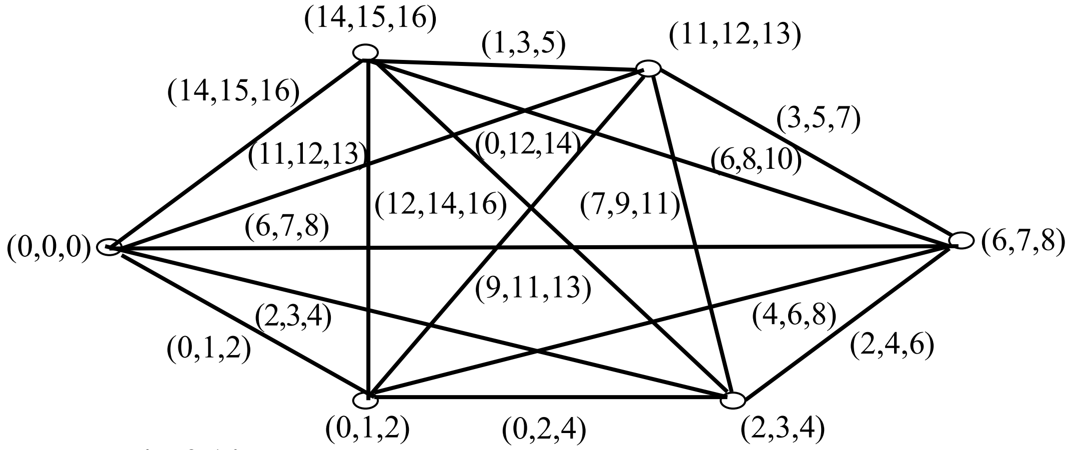

Fig. 2.14.

Remark 2.17. In crisp graph, K_6 is not a graceful complete graph.

Proposition 2.18. $_F C_{om}G$ K_7 is $_{FN}Gf$ complete graph.

Proof. From the $_F G$ figure, K_7 is $_{FN}Gf$ complete graph.

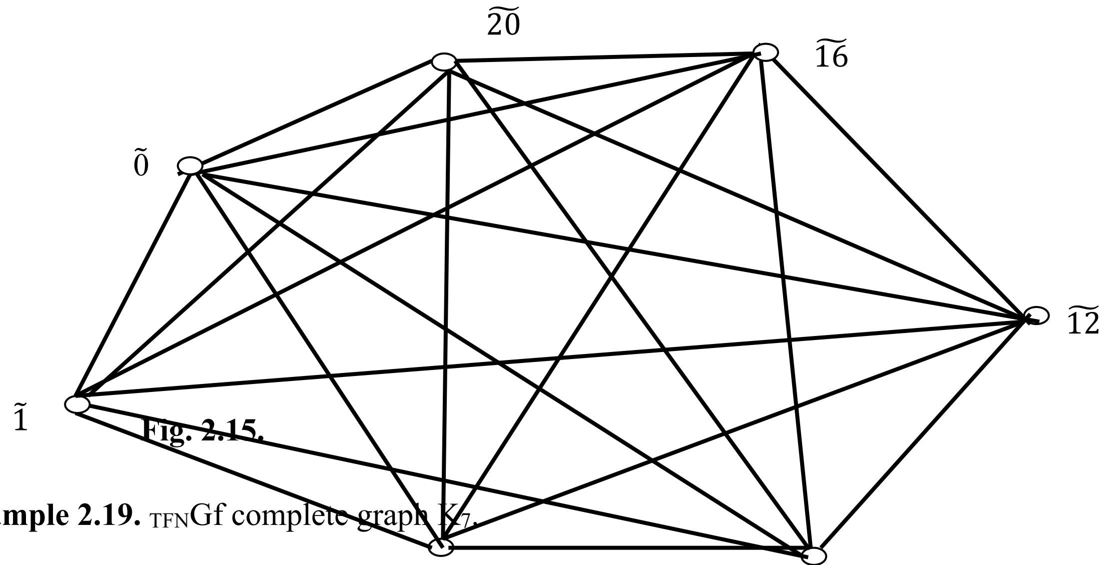

Fig. 2.15.

Example 2.19. $_{TFN}Gf$ complete graph K_7.

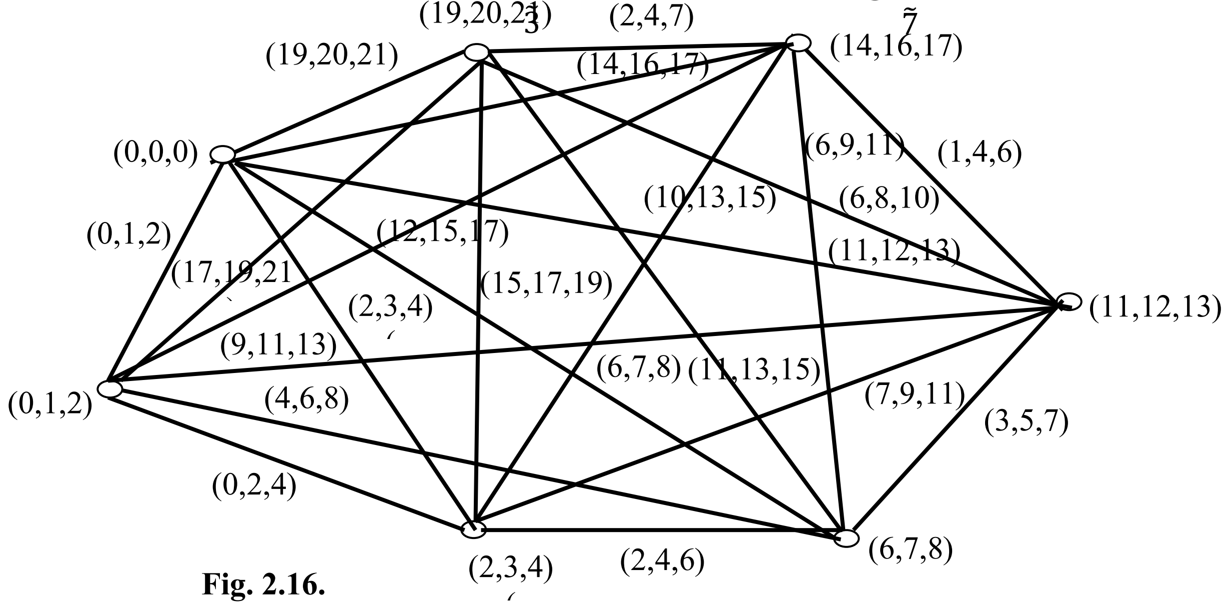

Fig. 2.16.

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Remark 2.20. In crisp graph, K_7 is not graceful complete graph.

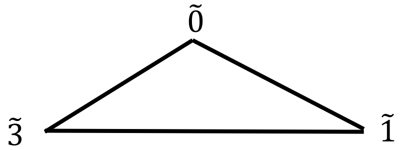
Proposition 2.21. Every $_F C_{om} G$ $\Omega = (\Phi, \Theta, \mathfrak{J})$ is $_{FN}$Gf complete graph.

Proof. From the generalization of the above proposition, Ω is a $_{FN}$Gf complete graph.

Proposition 2.22. $_F C_y$ C_3 is $_{FN}$Gf cycle graph.

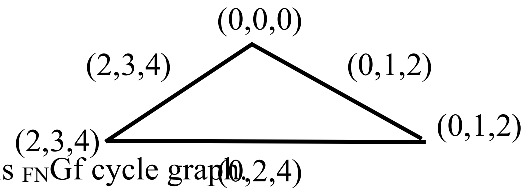
Proof. From the $_F$G figure, C_3 is $_{FN}$Gf cycle graph.

Fig. 2.17.



Example 2.23. $_{TFN}$Gf cycle graph C_3.

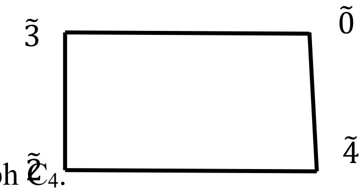
Fig. 2.18.



Proposition 2.24. $_F C_y$ C_4 is $_{FN}$Gf cycle graph.

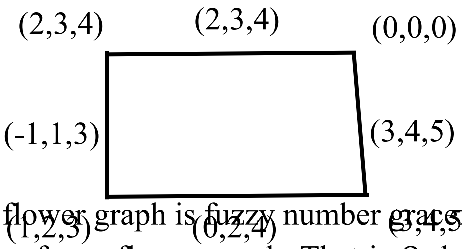
Proof. From the $_F$G figure, C_4 is $_{FN}$Gf cycle graph.

Fig. 2.19.



Example 2.25. $_{TFN}$Gf cycle graph C_4.

Fig. 2.20.



Theorem 2.26. Every fuzzy flower graph is fuzzy number graceful graph.

Proof. Let $\Omega = (\Phi, \Theta, \mathfrak{J})$ be a fuzzy flower graph. That is Ω has p fuzzy points and p-1 fuzzy edges.

Let $\Phi = \{(\mathfrak{w}_0, p_0), (\mathfrak{w}_1, p_1), \dots (\mathfrak{w}_{p-1}, p_{p-1})\}$ and $\Phi = \{(\mathfrak{y}_1, q_1), (\mathfrak{y}_2, q_2), \dots (\mathfrak{y}_{p-1}, q_{p-1})\}$ be fuzzy points and fuzzy edges.

A fuzzy flower graph exactly only one fuzzy points is adjacent with all the remaining (p-1) fuzzy points. Suppose that the fuzzy point $(\mathfrak{w}_0, p_0)$ is adjacent with all the remaining p-1 fuzzy points, then the mapping $\mathcal{L}$ is defined as $\mathcal{L}((\mathfrak{w}_0, p_0), (\mathfrak{w}_j, p_j)) = \tilde{j}, \tilde{j} = \tilde{1}, \tilde{2}, \dots \widetilde{p} - 1$, clearly all are distinct, so Ω is a fuzzy number graceful graph.

Remark 2.27. Every crisp graceful flower graph is fuzzy number graceful flower graph.

Example 2.28. A triangle fuzzy number flower graph with 6 petals is given below.

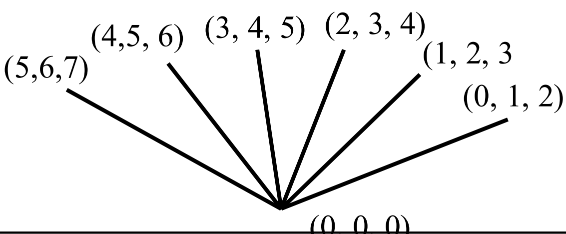


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Fig. 2.21. A triangle fuzzy number Graceful flower graph

Theorem 2.29. Every fuzzy star graph is fuzzy number graceful graph.

Proof. Let $\Omega = (\Phi, \Theta, \mathfrak{F})$ be a fuzzy star graph. That is Ω has p fuzzy points and p-1 fuzzy edges. Let $\Phi = \{(\mathfrak{w}_0, p_0), (\mathfrak{w}_1, p_1), \ldots (\mathfrak{w}_{p-1}, p_{p-1})\}$ and $\Phi = \{(\mathfrak{y}_1, q_1), (\mathfrak{y}_2, q_2), \ldots (\mathfrak{y}_{p-1}, q_{p-1})\}$ be fuzzy points and fuzzy edges.

A fuzzy star graph exactly only one fuzzy points is adjacent with all the remaining (p-1) fuzzy points. Suppose that the fuzzy point $(\mathfrak{w}_0, p_0)$ is adjacent with all the remaining p-1 fuzzy points, then the mapping $\mathcal{L}$ is defined as $\mathcal{L}((\mathfrak{w}_0, p_0), (\mathfrak{w}_j, p_j)) = \tilde{j}, \tilde{j} = \tilde{1}, \tilde{2}, \ldots \tilde{p} - 1$, clearly all are distinct, so Ω is a fuzzy number graceful graph.

Remark 2.30. Every crisp graceful star graph is fuzzy number graceful star graph.

Example 2.31. A triangle fuzzy number star graph with 8 petals is given below.

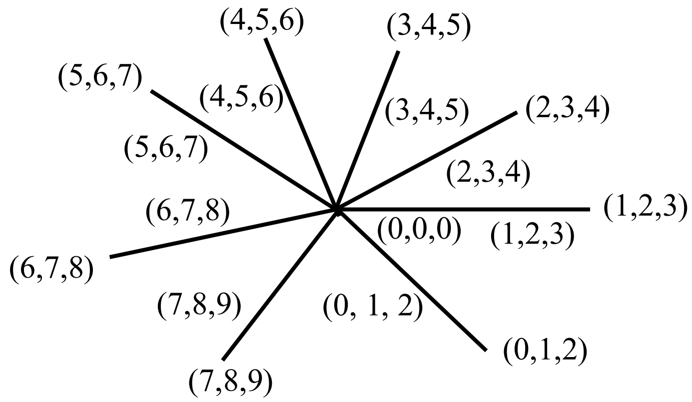
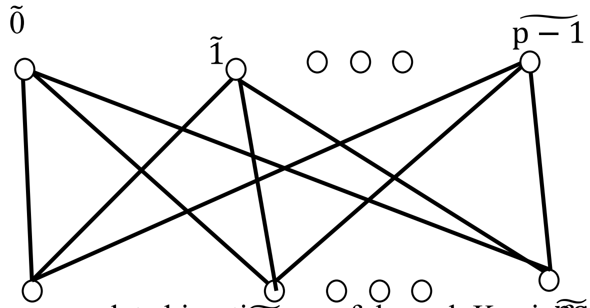


Fig. 2.22. A triangle fuzzy number Graceful star graph

Theorem 2.32. Every fuzzy complete bipartite graph $K_{p,q}$ is fuzzy number graceful graph.

Proof. Let $\Omega = (\Phi, \Theta, \mathfrak{F})$ be a fuzzy bipartite graph with p fuzzy points in one partition and q fuzzy points in another partition. Assign $\tilde{0}, \tilde{1}, \tilde{2}, \ldots, \tilde{p} - 1$ in p fuzzy points and assign $\tilde{p}, \widetilde{2p}, \ldots, \widetilde{pq}$ in q fuzzy points. Then the each fuzzy edge has distinct fuzzy number. Hence Ω is a fuzzy number graceful graph.

Fig. 2.23.



Remark 2.33. Every crisp complete bipartite graceful graph $K_{p,q}$ is fuzzy number complete bipartite graceful graph.

Example 2.34. A triangle fuzzy number complete bipartite graph $K_{3,4}$ is given below.

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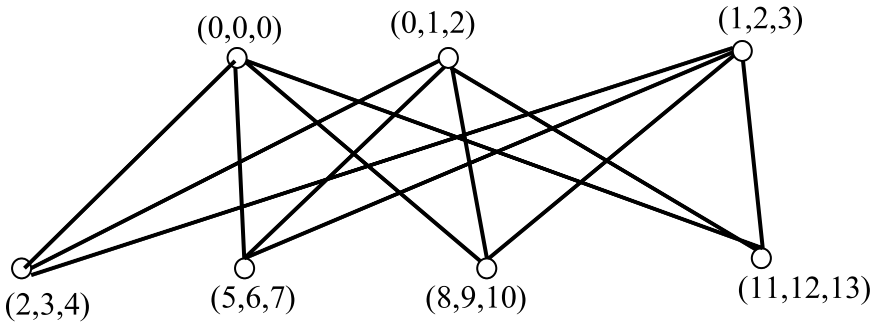

Fig. 2.24. A triangle fuzzy number Graceful complete bipartite graph

CONCLUSION

Fuzzy number graceful labeling is one of the main branch in fuzzy graph. Here different types of graceful labeling definition is given, and one or two simple theorem is given, these definitions are defined based on the crisp graph. Using the above definition and theorems, we can find more results. It can be extended into different types of fuzzy number graceful labeling.

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