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AN APPROACH OF SMART TECHNIQUE IN PICTURE FUZZY SCHWEIZER- SKLAR POWER GEOMETRIC AGGREGATION OPERATORS FOR DE- VELOPING ROAD CONSTRUCTION COMPANIES WITH GROUP DECISION- MAKING

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Abstract. The objective of this paper is to develop picture fuzzy aggregation operators by utilizing the concept of power aggregation operators through Schweizer-Sklar operations. The Schweizer-Sklar t-norm and t-conorm enhance the flexibility of the data integration process, as well as the power aggregation operator, by capturing the interrelationships between various criteria during decision making. Motivated by Schweizer-Sklar t-norm and t-conorm, this paper aims to develop the theory of the picture fuzzy Schweizer-Sklar power weighted geometric operator and the picture fuzzy Schweizer-Sklar power ordered weighted geometric operator. The paper also explores the properties and characteristics of these proposed operators. Criteria weights play a crucial role in aggregating different criteria in multiple criteria decision making processes. This work adopts the simple multi attribute rating technique to compute criteria weights for solving Multi-criteria group decision-making problems in a picture fuzzy environment. Finally, an illustrative example of road construction company is provided to demonstrate the applicability of the proposed operators. A comparison with existing operators validates the effectiveness of the proposed operators.

Keywords: Multi-criteria group decision-making, picture fuzzy set, Schweizer-Sklar t-norms and t-conorms, Power aggregation operator, Road construction companies.

AMS Subject Classification: 94D05, 91B06, 47S40.

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1 Introduction

Multi-criteria group decision-making (MCGDM) is an important area of decision theory with widespread applica- tions across various fields. In many cases, decision-makers encounter difficulties in reaching reasonable conclusions, as the process involves identifying multiple criteria and evaluating several alternatives. Moreover, in real-world decision-making, the ambiguity and subjectivity inherent in human qualitative judgments often result in expert opinions that encompass a range of responses—such as yes, abstain, no, or refusal—which cannot be accurately represented by crisp values. In 1965, Zadeh (1965) introduced the foundational concept of fuzzy set (FS) theory, which incorporated the degree of membership μ. This concept has since been applied to numerous real-world problems. Later, in Atanassov (1986) expanded on FS theory by developing the intuitionistic fuzzy set (IFS) theory, which adds the degree of non-membership ν alongside the degree of membership μ under the condition that $0 \leq \mu + \nu \leq 1$.

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The IFS, while demonstrating strong capabilities in multi criteria decision-making (MADM), cannot ade- quately express more complex fuzzy information. One limitation is its inability to capture ambiguous informa- tion, such as the degree of neutrality (η). To address this issue, Cuong et al. (2013), introduced the concept of the picture fuzzy set (PFS), which extends IFS by adding the degree of neutrality (η) alongside the degrees of membership (μ) and non-membership (v), under the condition $0 \leq \mu + \eta + v \leq 1$. PFS models are particularly useful in situations where human opinions involve more nuanced responses such as "yes," "no," "abstain," and "refusal." The figure 1 illustrates the extensions of a PFS. Additionally, a few students may express refusal to visit either destination (π). Cuong et al. (2016) investigated the classification of representable picture t-norms and picture t-conorms along with their properties. Yager (1988) introduced the ordered weighted aggregation (OWA) operator and explored its properties. Wei (2017) described various PF arithmetic and geometric operators, as well as PF hybrid aggregation operators (AOs) and their applications. Garg (2017) presented a series of AOs for PFSs that were applied to solve multi criteria decision-making (MCDM) problems. Wei (2018) studied the MADM problem using arithmetic and geometric AOs based on Hamacher operations in the PF context. Khan

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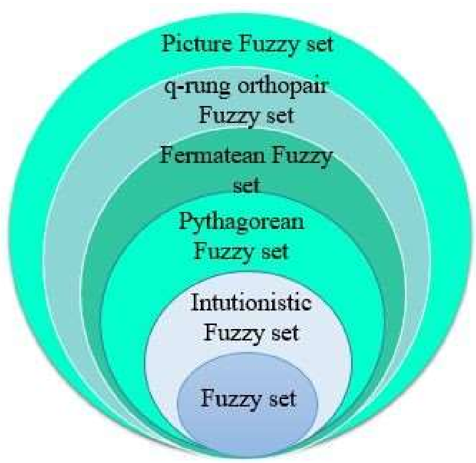

Figure 1: Extensions of PFS

et al. (2019) proposed AOs using PF Einstein operations and discussed their application in a PF-based MADM problem. Zhang et al. (2018) introduced PF Dombi Heronian mean operators for the MADM problem. Jana et al. (2019) developed PF Dombi arithmetic and geometric AOs to solve MADM problems. Additionally, Jana et al. (2019) employed PF Hamacher AOs to construct MADM approaches for enterprise performance evaluation within the PF context. Ates et al. (2020) proposed novel PF aggregation operators and extended the Bonferroni mean operator to MCDM applications. Wang et al. (2018) proposed PF Muirhead mean operators and demonstrated their application for financial investment risk evaluation. Qiyas et al. (2020) introduced several AOs using Yager operators in the PF context. Several other MADM approaches based on AOs for PFNs have been proposed, including those by Lin et al. (2021), Qin et al. (2021), Seikh et al. (2021), Ullah (2021), and Senapati (2022).

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Table 1: Comparison of different approaches for picture fuzzy AOs

Authors	Geometric operator	Status of criteria weight	DM method	DM solution
Garg, et al. (2023)	IF Schweizer-Sklar power IF Schweizer-Sklar weighted	Known	MADM	Score function
Khan et al. (2022)	IF Schweizer-Sklar generalized power weighted IF Schweizer-Sklar generalized power ordered weighted	Known	MAGDM	Score function
Biswas, A & Deb, N. (2021)	PyF Schweizer-Sklar power weighted	Known	MADM	Score function
Wei, D et al. (2022)	FF Schweizer-Sklar weighted	Unknown	MAGDM	Best-and-worst
Ma et al. (2024)	q-ROF Schweizer-Sklar power q-ROF Schweizer-Sklar power weighted	Known	MADM	Score function
Hussain et al (2022)	PF Schweizer-Sklar prioritized PF Schweizer-Sklar prioritized weighted	Known	MADM	Score function
Proposed work	PF Schweizer-Sklar power weighted PF Schweizer-Sklar power ordered weighted	Unknown	MAGDM	Score function

Furthermore, various AO based on the Schweizer-Sklar (SS) operational law within different FS frameworks are summarized in Table 1. Harish Garg, et al. (2023) introduced the IFSS prioritized AO for solving MCDM problems. Khan et al. (2022) explored the concept of the IFSS generalized power AO. Biswas, A & Deb, N. (2021) developed the SS power AO in a pythagorean fuzzy (PyF) environment. Wei, D et al. (2022) established the fermatean fuzzy (FF) SS in green supplier selection. Ma et al.(2024) proposed the q-rung orthopair fuzzy (q-ROF) SS prioritized operator for MADM problems. Hussain et al (2022) developed the PFSS prioritized op- erator for solving MADM problems.

This article introduces a series of efficient AO by combining the power aggregation (PA) operator with SS op- erations to enhance the information aggregation process in the PF environment. A set of PF AO is proposed, including the PFSS power geometric (PFSSPG) operator, PFSS power weighted geometric PFSSPWG operator, and PFSS power ordered weighted geometric (PFSSPOWG) operator. The article also discusses special cases and desirable properties of these operators. A simple

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multi criteria rating technique (SMART) is presented to solve MCGDM problems in the PF environment using the proposed operators. To demonstrate the practicality and effectiveness of the approach, an illustrative example, is solved and compared with several existing approaches. Additionally, Table 1 provides a summary of various AO based on the SS operational law. Our study aims to address this specific research gap.

The main motivations of this article are as follows:

- The previous research focuses on the development of AO, with particular emphasis on the importance of SS operations, which serve as a generalization of algebraic operations.
- To combine decision information from multiple experts using an PG operator that accounts for the inter- relationships between criteria.
- To develop an MCGDM method incorporating the proposed operators for efficiently managing conflicting criteria.
- To showcase the practicality of the proposed operators through solving a numerical MCGDM application.
- To validate the feasibility of the method by examining the parameters of the defined operators across different values and comparing them with existing operators.

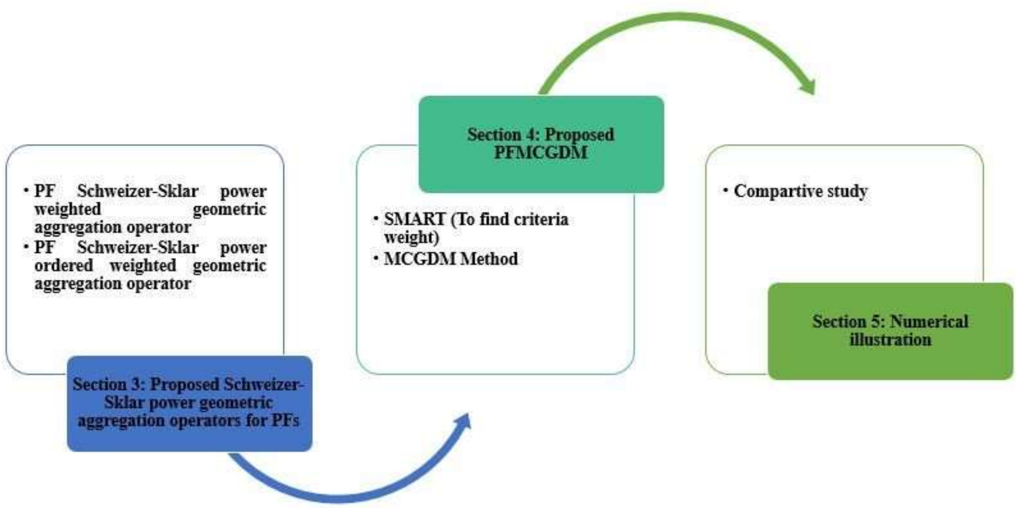

Figure 2: Contribution of the proposed work

The main contributions of this article are as follows:

- This article introduces innovative SS operation laws and PA operator, namely PFSSPG, PFSSPWG and PFSSPOWG operators under PF environment.
- These operators are utilized to develop a novel PFMCGDM method with SMART.
- The effectiveness and reliability of the proposed PFMCGDM method with SMART approach are demon- strated by applying it to solve a practical road construction companies.
- Furthermore, the feasibility of the developed method is illustrated through a comparative evaluation against existing operators.
- The contributions of this research are illustrated in Figure 2 to provide readers with a clear visual under- standing.

This research work is organized into eight detailed sections as outlined below. Section 2 introduces the fundamental concept of PFSs and their related operations. In Section 3, the proposed picture fuzzy Schweizer- Sklar power geometric aggregation operators, along with their properties, are discussed. Section 4 continues this discussion by presenting the properties of these aggregation operators. Section 5 outlines an algorithm to solve MCGDM problems using the proposed operators. To illustrate the practical application of this approach, a numerical example in the picture fuzzy context is provided in Section 6. Section 7 offers a comparative analysis to demonstrate the advantages of the proposed method over existing aggregation operators. Finally, Section 8 presents the conclusions drawn from the study.

2 Preliminaries

In this section, some basic concepts have been reviewed related to PFS.

2.1 Picture Fuzzy Set

The PFS (Cuong et al. (2013, 2014)) is an extension of IFS. The mathematical form of PFS is expressed as follows:

Definition 1. *A picture fuzzy set A on universal set X is defined by,*

$$A = \{\langle x, \mu A(x), \eta A(x), \nu A(x)\rangle / x \in X\}$$

Where, $\mu\mathrm{A}(x)$, $\eta\mathrm{A}(x)$ *and* $\nu\mathrm{A}(x) \in [0, 1]$ *are the degree of membership, the degree of neutral membership and the degree of non-membership of* $x \in \mathrm{A}$ *respectively, with the following condition:* $0 \leq \mu\mathrm{A}(x) + \eta\mathrm{A}(x) + \nu\mathrm{A}(x) \leq 1$ $\forall x \in X$. *Then, for* $x \in X$, $\pi\mathrm{A}(x) = 1 - \mu\mathrm{A}(x) - \eta\mathrm{A}(x) - \nu\mathrm{A}(x)$ *could be called the degree of refusal membership of x in* A. *Geometrical representation of PFS is shown in Figure 1. For convenience,* $\alpha = (\mu\alpha, \eta\alpha, \nu\alpha)$ *is called a picture fuzzy number (PFN).*

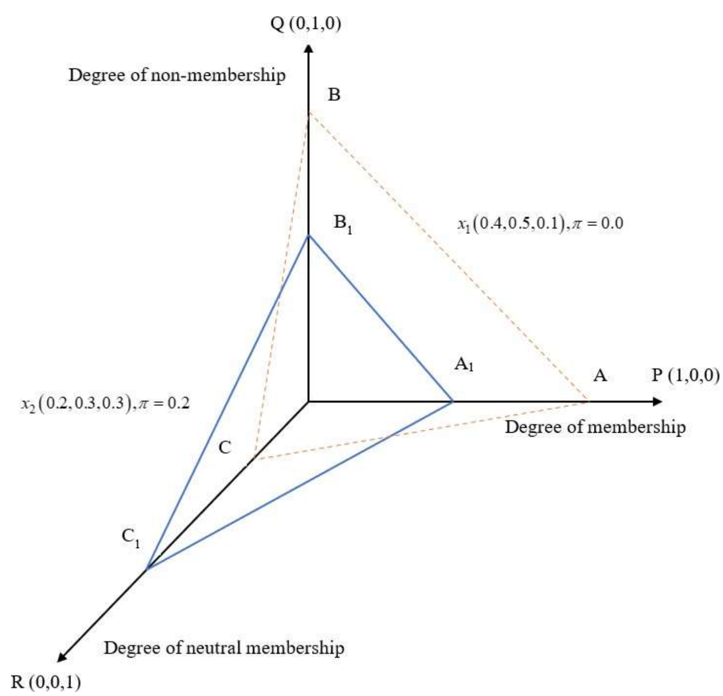

Figure 3: Geometrical representation of picture fuzzy set

2.2 Comparison for PFNs

According to Garg (2017) the score and accuracy functions of PFNs are as follows:

Definition 2. *Let* $\alpha = (\mu\alpha, \eta\alpha, \nu\alpha)$ *be a PFN, its score function* $S(\alpha)$ *and its accuracy function* $A(\alpha)$ *is defined by* $S(\alpha) = \mu\alpha - \eta\alpha - \nu\alpha$; $S(\alpha) \in [-1, 1]$, $A(\alpha)$

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$= \mu_\alpha + \eta_\alpha + \nu_\alpha$; $A(\alpha) \in [0, 1]$.

Based on the $S(\alpha)$ and $A(\alpha)$ an order relationship between two PFNs is defined as follows.

Definition 3. *Let $\alpha_1 = (\mu_{\alpha_1}, \eta_{\alpha_1}, \nu_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, \eta_{\alpha_2}, \nu_{\alpha_2})$ be two PFNs. Then the following comparison rules can be used:*

(i) *If $S(\alpha_1) < S(\alpha_2)$ then $\alpha_1 \prec \alpha_2$*

(ii) *If $S(\alpha_1) = S(\alpha_2)$ then*

  (a) *If $A(\alpha_1) < A(\alpha_2)$ then $\alpha_1 \prec \alpha_2$*

  (b) *If $A(\alpha_1) = A(\alpha_2)$ then $\alpha_1 \approx \alpha_2$.*

2.3 Operation laws of Picture fuzzy numbers

Schweizer-Sklar (Hussain, A. et al. (2022)) operations laws and power geometric aggregation operator (Ullah, K. et all (2023)) are defined for PFNs as follows.

Definition 4. *The PG operator are defined as:*

$$PG(\breve{b}_1, \breve{b}_2, \ldots, \breve{b}_r) = \prod_{k=1}^{r} \breve{b}_k^{\frac{(1+\ddot{U}(\breve{b}_k))}{\sum_{k=1}^{r} 1+\ddot{U}(b_k)}} \tag{1}$$

where $\ddot{U}(\breve{b}_k) = \sum_{k=1, k\neq h}^{r} sup(\breve{a}_k, \breve{a}_h)$, $sup(\breve{a}_k, \breve{a}_h) = 1 - D(\breve{a}_k, \breve{a}_h)$ and the weight $\frac{(1+\ddot{U}(\breve{b}_k))}{\sum_{k=1}^{r} 1+\ddot{U}(b_k)}$ of the argument b_h depends on all the input arguments $\breve{b}_k (k = 1, 2, \ldots, r)$, which enables the argument values to support each other in the geometric aggregation process.

Definition 5. *Let $\alpha_1 = \langle \mu_{\alpha_1}, \eta_{\alpha_1}, \nu_{\alpha_1} \rangle$ and $\alpha_2 = \langle \mu_{\alpha_2}, \eta_{\alpha_2}, \nu_{\alpha_2} \rangle$ be any two PFNs, then the Euclidean distance between them is defined as follows:*

$$D(\alpha_1, \alpha_2) = \frac{1}{3}\{|\mu_{\alpha_1} - \mu_{\alpha_2}| + |\eta_{\alpha_1} - \eta_{\alpha_2}| + |\nu_{\alpha_1} - \nu_{\alpha_2}|\} \tag{2}$$

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Definition 6. *Let $\alpha_1 = \langle \mu_{\alpha_1}, \eta_{\alpha_1}, \nu_{\alpha_1} \rangle$ and $\alpha_2 = \langle \mu_{\alpha_2}, \eta_{\alpha_2}, \nu_{\alpha_2} \rangle$ be any two PFN. Then the generalized union and intersection are defined as follows:*

$\alpha_1 \cup \alpha_2 = \{\langle x, \{T^*\{\mu_{\alpha_1}(x), \mu_{\alpha_2}(x)\}\}, \{T\{\eta_{\alpha_1}(x), \eta_{\alpha_2}(x)\}\}, \{T\{\nu_{\alpha_1}(x), \nu_{\alpha_2}\}(x)\} | x \in X\rangle\}$;

$\alpha_1 \cap \alpha_2 = \{\langle x, \{T\{\mu_{\alpha_1}(x), \mu_{\alpha_2}(x)\}\}, \{T^*\{\eta_{\alpha_1}(x), \eta_{\alpha_2}(x)\}\}, \{T^*\{\nu_{\alpha_1}(x), \nu_{\alpha_2}(x)\}\} | x \in X\}\rangle\}$; *where* T *and* T^* *respectively, express TN and TCN.*

The SSTN and SSTCN are defined as follow.

$$T(x_1, x_2) = (x_1^{\beta} + x_2^{\beta} - 1)^{\frac{1}{\beta}}$$

$$T^*(x_1, x_2) = 1 - ((1 - x_1)^{\beta} + (1 - x_2)^{\beta} - 1)^{\frac{1}{\beta}}$$

Additionally, when β=0, we have $T(x_1, x_2) = x_1 x_2$ *and* $T^*(x_1, x_2) = x_1 + x_2 - x_1 x_2$. *That is, SSTN and SSTCN reduce to algebraic TN and TCN.*

Definition 7. *Let $\alpha_1 = \langle \mu_{\alpha_1}, \eta_{\alpha_1}, \nu_{\alpha_1} \rangle$ and $\alpha_2 = \langle \mu_{\alpha_2}, \eta_{\alpha_2}, \nu_{\alpha_2} \rangle$ be any two PFN. Then based on SS operations, the generalized union and intersection are introduced as follows:*

$$\alpha_1 \oplus \alpha_2 = \langle T^*(\mu_{\alpha_1}, \mu_{\alpha_2}), T(\eta_{\alpha_1}, \eta_{\alpha_2}), T(\nu_{\alpha_1}, \nu_{\alpha_2})\rangle \quad (3)$$

$$\alpha_1 \otimes \alpha_2 = \langle T(\mu_{\alpha_1}, \mu_{\alpha_2}), T^*(\eta_{\alpha_1}, \eta_{\alpha_2}), T^*(\nu_{\alpha_1}, \nu_{\alpha_2})\rangle \quad (4)$$

Definition 8. *Let $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$, $\alpha_1 = (\mu_{\alpha_1}, \eta_{\alpha_1}, \nu_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, \eta_{\alpha_2}, \nu_{\alpha_2})$ be three PFNs and some basic algebraic operations of PFNs are defined as follow:*

$$\alpha_1 \oplus \alpha_2 = \langle \mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1}\mu_{\alpha_2}, \eta_{\alpha_1}\eta_{\alpha_2}, \nu_{\alpha_1}\nu_{\alpha_2}\rangle,$$

$$\alpha_1 \otimes \alpha_2 = \langle \mu_{\alpha_1}\mu_{\alpha_2}, \eta_{\alpha_1} + \eta_{\alpha_2} - \eta_{\alpha_1}\eta_{\alpha_2}, \nu_{\alpha_1} + \nu_{\alpha_2} - \nu_{\alpha_1}\nu_{\alpha_2}\rangle,$$

$$\lambda\alpha = \langle 1 - (1 - \mu_\alpha)^\lambda, (\eta_\alpha)^\lambda, (\nu_\alpha)^\lambda\rangle,$$

$$\alpha^\lambda = \left\langle (\mu_\alpha)^\lambda, 1 - (1 - \eta_\alpha)^\lambda, 1 - (1 - \nu_\alpha)^\lambda \right\rangle : \lambda > 0.$$

Definition 9. *Let $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$, $\alpha_1 = (\mu_{\alpha_1}, \eta_{\alpha_1}, \nu_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, \eta_{\alpha_2}, \nu_{\alpha_2})$ be three PFNs and some basic SS operations of PFNs are defined as follows ($\beta < 0$):*

$$\alpha_1 \oplus \alpha_2 = \langle 1 - ((1 - \mu_{\alpha_1})^{\beta} + (1 - \mu_{\alpha_2})^{\beta} - 1)^{\frac{1}{\beta}}, (\eta_{\alpha_1}^{\beta} + \eta_{\alpha_2}^{\beta} - 1)^{\frac{1}{\beta}}, (\nu_{\alpha_1}^{\beta} + \nu_{\alpha_2}^{\beta} - 1)^{\frac{1}{\beta}}\rangle,$$

$$\alpha_1 \otimes \alpha_2 = \langle (\mu_{\alpha_1}^{\beta} + \mu_{\alpha_2}^{\beta} - 1)^{\frac{1}{\beta}}, 1 - ((1 - \eta_{\alpha_1})^{\beta} + (1 - \eta_{\alpha_2})^{\beta} - 1)^{\frac{1}{\beta}}, 1 - ((1 -$$

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$v\alpha 1$) $+ (1 - v\alpha 2$) $- 1)^{\beta} \rangle,$
1 2 λ - λ $\frac{1}{}$
λ - $\frac{1}{}$
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$\alpha_1 = \langle(\lambda\mu\ \beta - (\lambda - 1))^{\beta}, 1 - (\lambda(1 - \eta\alpha 1)$ $- (\lambda - 1))^{\beta}, 1 - (\lambda(1 - v\alpha 1) - (\lambda - 1))^{\beta} \rangle, \lambda > 0,$
1

λ $\frac{1}{}$ $\frac{1}{}$ $\frac{1}{}$
$\lambda\alpha 1 = \langle 1 - (\lambda(1 - \mu\alpha 1) - (\lambda - 1))^{\beta}, (\lambda\eta_{\alpha}\beta - (\lambda - 1))^{\beta}, (\lambda v_{\alpha}\beta - (\lambda - 1))^{\beta} \rangle, \lambda > 0.$
1 1

3 Picture fuzzy Schweizer-Sklar power geometric aggregation operators

In this section, we presented the Schweizer-Sklar power geometric AOs and discussed the fundamental properties of these proposed operators within the framework of PFNs.

3.1 Picture fuzzy Schweizer-Sklar power weighted geometric aggregation op- erators

In the next section, we introduce the confidence picture fuzzy Schweizer-Sklar power weighted geometric operator within the context of PF, and we examine the desirable properties of this proposed operator. Additionally, we applied a weighted suppport degree throughout our article, using the following equation: $\breve{\chi}_j$ ($j = 1, 2, \ldots, n$) is a set of integrated weights, $\breve{\chi}_j = \frac{w_j(1+\mathrm{Ny}(\alpha_j))}{\sum_{j=1}^{n} w_j(1+\mathrm{Ny}(\alpha_j))}$. $\mathrm{Ny}(\alpha_j) = \sum_{h=1, h\neq j}^{n} Sup(\alpha_j, \alpha_h)$ and $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $\alpha_j (j = 1, 2, \ldots, n) w \in [0, 1], \sum_{j=1}^{n} w_j = 1.$

Definition 10. *Let $\alpha_j = \langle \mu_{\alpha_j}, \eta_{\alpha_j}, v_{\alpha_j} \rangle (j = 1, 2, \ldots, n)$ be a set of PFN. Then the PFSSPWG operator of dimension n is a mapping PFSSPWG: $\alpha^n \to \alpha$ such that*

$$PFSSPWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigoplus_{j=1}^{n} \alpha_j^{\left(\frac{w_j(1+\mathrm{Ny}(\alpha_j))}{\sum_{j=1}^{n} w_j(1+\mathrm{Ny}(\alpha_j))}\right)} \quad (5)$$

where α is the set of all PFN and $\mathrm{Ny}(\alpha_j) = \sum_{h=1, h\neq j}^{n} Sup(\alpha_j, \alpha_h)$ and $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of α_j ($j = 1, 2, \ldots, n$)$w \in [0, 1], \sum_{j=1}^{n} w_j = 1.$

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Theorem 1. *Let $\alpha_j = \langle \mu_{\alpha_j}, \eta_{\alpha_j}, \nu_{\alpha_j} \rangle (j = 1, 2, \ldots, n)$ be a set of PFN and $\beta < 0$, then the aggregated value obtained using $PFSSPWG$ operator is also a PFN and can be expressed as follows:*

$$PFSSPWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \langle \Big(\sum_{j=1}^{n} \breve{\chi}_j \mu_j^{\beta}\Big)^{\frac{1}{\beta}}, 1 - \Big(\sum_{j=1}^{n} \breve{\chi}_j (1-\eta_j)^{\beta}\Big)^{\frac{1}{\beta}}, 1 - \Big(\sum_{j=1}^{n} \breve{\chi}_j (1-\nu_j)^{\beta}\Big)^{\frac{1}{\beta}} \rangle \tag{6}$$

where $\breve{\chi}_j$ $(j = 1, 2, \ldots, n)$ is a set of integrated weights, $\breve{\chi}_j = \dfrac{1+Ny(\alpha_j)}{\sum_{j=1}^{n}(1+Ny(\alpha_j))}$ ·

In order to prove Equation 6, we first prove that when $\breve{\chi} = (\breve{\chi}_1, \breve{\chi}_2, \ldots, \breve{\chi}_n)^T$ is any value, i.e. without any con- straint for $\breve{\chi}$, the following equation is right: $PFSSPWG(\alpha_1, \alpha_2, \ldots, \alpha_n) =$

$$\langle 1 - \Big(\sum_{j=1}^{n} \breve{\chi}_j (1-\mu_j)^{\beta} - \sum_{j=1}^{n} \breve{\chi}_j + 1\Big)^{\frac{1}{\beta}}, \Big(\sum_{j=1}^{n} \breve{\chi}_j \eta_j^{\beta} - \sum_{j=1}^{n} \breve{\chi}_j + 1\Big)^{\frac{1}{\beta}}, \Big(\sum_{j=1}^{n} \breve{\chi}_j \nu_j^{\beta} - \sum_{j=1}^{n} \breve{\chi}_j + 1\Big)^{\frac{1}{\beta}} \rangle \tag{7}$$

By the mathematical induction method, the Equation 7 can be proved as follows:

Proof. According to the operational rules of PFs based on SS operations, we have

$$\breve{\chi}_j \alpha_j = \langle (\breve{\chi}_j \mu_j^{\beta} - \breve{\chi}_j + 1)^{\frac{1}{\beta}}, 1 - (\breve{\chi}_j (1-\eta_j)^{\beta} - \breve{\chi}_j + 1)^{\frac{1}{\beta}}, 1 - (\breve{\chi}_j (1-\nu_j)^{\beta} - \breve{\chi}_j + 1)^{\frac{1}{\beta}} \rangle$$

1. When n=2, we have $\breve{\chi}_1 \alpha_1 = \langle (\breve{\chi}_1 \mu_1^{\beta} - \breve{\chi}_1 + 1)^{\frac{1}{\beta}}, 1 - (\breve{\chi}_1 (1-\eta_1)^{\beta} - \breve{\chi}_1 + 1)^{\frac{1}{\beta}}, 1 - (\breve{\chi}_1 (1-\nu_1)^{\beta} - \breve{\chi}_1 + 1)^{\frac{1}{\beta}} \rangle$;

   $\breve{\chi}_2 \alpha_2 = \langle (\breve{\chi}_2 \mu_2^{\beta} - \breve{\chi}_2 + 1)^{\frac{1}{\beta}}, 1 - (\breve{\chi}_2 (1-\eta_2)^{\beta} - \breve{\chi}_2 + 1)^{\frac{1}{\beta}}, 1 - (\breve{\chi}_2 (1-\nu_2)^{\beta} - \breve{\chi}_2 + 1)^{\frac{1}{\beta}} \rangle$

   then $PFSSPWG(\alpha_1, \alpha_2) =$

   $\breve{\chi}_1 \alpha_1 + \breve{\chi}_2 \alpha_2 = \langle (\breve{\chi}_1 \mu_1^{\beta} - \breve{\chi}_1 + 1)^{\frac{1}{\beta}}, 1 - (\breve{\chi}_1 (1-\eta_1)^{\beta} - \breve{\chi}_1 + 1)^{\frac{1}{\beta}}, 1 - (\breve{\chi}_1 (1-\nu_1)^{\beta} - \breve{\chi}_1 + 1)^{\frac{1}{\beta}} + (\breve{\chi}_2 \mu_2^{\beta} - \breve{\chi}_2 + 1)^{\frac{1}{\beta}}, 1 - (\breve{\chi}_2 (1-\eta_2)^{\beta} - \breve{\chi}_2 + 1)^{\frac{1}{\beta}}, 1 - (\breve{\chi}_2 (1-\nu_2)^{\beta} - \breve{\chi}_2 + 1)^{\frac{1}{\beta}} \rangle = \langle (\sum_{j=1}^{2} \breve{\chi}_j \mu_j^{\beta} - \sum_{j=1}^{2} \breve{\chi}_j + 1)^{\frac{1}{\beta}}, 1 - (\sum_{j=1}^{2} \breve{\chi}_j (1-\eta_j)^{\beta} - \sum_{j=1}^{2} \breve{\chi}_j + 1)^{\frac{1}{\beta}}, 1 - (\sum_{j=1}^{2} \breve{\chi}_j (1-\nu_j)^{\beta} - \sum_{j=1}^{2} \breve{\chi}_j + 1)^{\frac{1}{\beta}} \rangle$ i.e. when n=2, Equation 7 is right.

2. Suppose n=m, Equation 7 is right, i.e. $PFSSPWG(\alpha_1, \alpha_2, \ldots, \alpha_m) = \langle 1 - (\sum_{j=1}^{m} \breve{\chi}_j (1-\mu_j)^{\beta} - \sum_{j=1}^{m} \breve{\chi}_j + 1)^{\frac{1}{\beta}}, (\sum_{j=1}^{m} \breve{\chi}_j \eta_j^{\beta} - \sum_{j=1}^{m} \breve{\chi}_j + 1)^{\frac{1}{\beta}}, (\sum_{j=1}^{m} \breve{\chi}_j \nu_j^{\beta} - \sum_{j=1}^{m} \breve{\chi}_j + 1)^{\frac{1}{\beta}} \rangle$ then when n=m+1, according to the op-

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β Σ_m $\underline{1}$

erational rules of PFNs, we have $\breve{\chi}_{m+1}\alpha_{m+1} = \langle(\breve{\chi}_{m+1}\mu_{m+1} - \breve{\chi}_{m+1} + 1)^{\beta}$, 1 $-(\breve{\chi}_{m+1}(1-\eta_{m+1}) - \breve{\chi}_{m+1} +$

$\underline{1}$ β $\underline{1}$

$1)^{\beta}$, $1-(\breve{\chi}_{m+1}(1-v_{m+1}) - \breve{\chi}_{m+1}+1)^{\beta}\rangle$ and $PFSSPWG(\alpha_1, \alpha_2, \ldots, \alpha_m, \alpha_{m+1}) = PFSSPWG(\alpha_1, \alpha_2, \ldots, \alpha_m) \oplus$

$(\breve{\chi} \quad \alpha \quad) = \langle 1 - \quad \breve{\chi} \ (1-\mu\ )^{\beta}$ $\frac{1}{\Sigma_m}$ $\breve{\chi}\ \eta^{\beta} -$ $\frac{1}{\Sigma_m}$ $\breve{\chi}\ v^{\beta} -$

$(\Sigma_m$ $-\Sigma_m$ Σ_m

$m+1 \ m+$ $j=$ j $j=1\ \breve{\chi}_j$ $j=1$ $j=1\ \breve{\chi}_j +$ $j=1$

Σ_m 1 1 $+1)^{\beta}$, ($j\ j$ $1)^{\beta}$, (j

$j=1\ \breve{\chi}_j + 1)^{\beta} + (\breve{\chi}_{m+1}\mu_{m+1})^{\beta} \breve{\chi}_{m+1} + 1)^{\beta}\underline{1}$ $1-(\breve{\chi}_{m+1}(1-\eta_{m+1}))^{\beta} - \breve{\chi}_{m+1}+1)\underline{1}$, $1-(\breve{\chi}_{m+1}(1-v_{m+1})$

j

β

$\breve{\chi}$ 1 $\Sigma_{m+1}\ \breve{\chi}\ \mu^{\beta}$ 1 $\Sigma_{m+1}\ \breve{\chi}\ (1-\eta$ 1 $\Sigma_{m+1}\ \breve{\chi}\ (1-$

$-\Sigma_{m+1}$ $)^{\beta}\ -\Sigma_{m+1}$

$m+1 \ +1)$ $j=1$ $j\ \ \breve{\chi}_j+1)^{\beta}$ $j=1$ $j=1\ \breve{\chi}_j+1)^{\beta}$, $j=1$ j

$^{\beta}\rangle = \langle($ j $= 1-($ j $1-($

$v_j)^{\beta} -_{j=1}$ j 1 j

Σ_{m+1} $\breve{\chi}_j + 1)^{\beta}\underline{1}$ So, when n=m+1, Equation 7 is right.

3. According to steps (1) and (2), Equation 7 is established for any j. Because Equation 7 is right without any constraint for $\breve{\chi}$, Equation 6 is also right when $\breve{\chi}_j \geq 0$ and $\Sigma^n\ \breve{\chi}_j = 1$.

□

Example 1. *Suppose* $\alpha_1 = \langle 0.8, 0.1, 0.05\rangle$, $\alpha_2 = \langle 0.52, 0.3, 0.1\rangle$, $\alpha_3 = \langle 0.45, 0.4, 0.01\rangle$ *and* $\alpha_4 = \langle 0.79, 0.156, 0.05\rangle$ *are four PFNs,* $w = (w_1, w_2, w_3, w_4) = \langle 0.2, 0.1, 0.3, 0.4\rangle^T$ *be the weight vector of* α_j $(j = 1, 2, 3, 4)$ *then we adopt the PFSSPWG to aggregate the four PFNs and generate a comprehensive value. The steps are shown as follows:* $(\beta = -2)$. $Sup(\alpha_1, \alpha_2) = 0.823$, $Sup(\alpha_1, \alpha_3) = 0.77$, $Sup(\alpha_1, \alpha_4) = 0.978$, $Sup(\alpha_2, \alpha_3) = 0.913$, $Sup(\alpha_2, \alpha_4) = 0.845$ *and* $Sup(\alpha_3, \alpha_4) = 0.792$.Ny$(\alpha_1) = Sup(\alpha_1, \alpha_2) + Sup(\alpha_3, \alpha_1) = 2.571$, Ny$(\alpha_2) = 2.582$, Ny$(\alpha_3) = 2.475$ *and* Ny$(\alpha_4) = 2.615$.$\breve{\chi}_1 = 0.201$, $\breve{\chi}_2 = 0.101$, $\breve{\chi}_3 = 0.293$ *and* $\breve{\chi}_4 = 0.406$. $PFSSPWG(\alpha_1, \alpha_2, \ldots, \alpha_4) = \langle 1 -$

$(\Sigma_{j=1}^{4}\breve{\chi}_j(1-\mu_j)^{\beta})^{\frac{1}{\beta}}$, $(\Sigma_{j=1}^{4}\breve{\chi}_j\eta_j)^{\beta})^{\frac{1}{\beta}}$, $(\Sigma_{j=1}^{4}\breve{\chi}_j v_j)^{\beta})^{\frac{1}{\beta}}\rangle = \langle 0.253, 0.262, 0.04\rangle$.

For a collection of PFN, we can easily prove the proposed PFSSPWG operator satisfying the idempotency, monotonicity and boundedness properties as follows:

property 1. *(Idempotency) Let* $\alpha_j = \langle\mu_{\alpha_j}, \eta_{\alpha_j}, v_{\alpha_j}\rangle (j = 1, 2, \ldots, n)$ *be a set of PFN, if* $\alpha_j = \alpha = (\mu, \eta, v)$ *then* $PFSSPWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha$

property 2. *(Monotonicity) Let* $\alpha = \langle\mu, \eta, v\rangle$ *and* $\alpha' = \langle\mu', \eta', v'\rangle$

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j αj *j* αj αj ⟩ $\alpha_j (j = 1, 2, \ldots, n)$ *as two*
′ ′ ′ αj *sets of*
αj αj

PFN; and $\mu j \leq \mu_j$, $\eta_j \leq \eta j$ *and* $\nu_j \leq \nu_j$ *then*

′ ′ ′

$$PFSSPWG(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq PFSSPWG(\alpha_1, \alpha_2, \ldots, \alpha_n)$$

property 3. *(Boundedness) Let* $\alpha j = \langle \mu\alpha j , \eta\alpha j , \nu\alpha j \rangle (j = 1, 2, \ldots, n)$ *be a set of PFN, and*

$\alpha^+ =$ $_{ij=1}$, $_{ij=1}$, $_{ij=1}$), $\alpha^- =$ $_{ij=1}$, $_{ij=1}$, $_{ij=1}$) *then*

$(max\mu^n$ $\underset{n}{min\eta}$ $\underset{n}{min\nu}$ $(min\mu^n$ $\underset{n}{max\eta}$ $max\nu$

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$$\alpha^{-} \leq PFSSPWG(\alpha1, \alpha2, \ldots, \alpha n) \leq \alpha^{+}$$

3.2 Picture fuzzy Schweizer-Sklar power ordered weighted geometric aggre- gation operators

In the following section, we propose the confidence picture fuzzy Schweizer-Sklar power ordered weighted geomet- ric operator with the PF context and the desirable properties of the proposed operator are investigated.

Definition 11. *Let* $\alpha j = \langle \mu\alpha j, \eta\alpha j, \nu\alpha j \rangle (j = 1, 2, \ldots, n)$ *be a set of SVNN. Then the PFSSPOWG operator of dimension n is a mapping PFSSPOWG:* $\alpha^n \rightarrow \alpha$ *such that*

$$PFSSPOWG(\alpha1, \alpha2, \ldots, \alpha n) = \bigoplus_{j=1}^{n} \alpha_{\sigma(j)}^{\left(\frac{wj(1+\mathrm{Ny}(\alpha_{\sigma(j)}))}{\sum_{j=1}^{n} wj(1+\mathrm{Ny}(\alpha_{\sigma(j)}))}\right)} \tag{8}$$

where $\sigma(j)$ *is the permutation such that* $\alpha_{\sigma(j-1)} \geq \alpha_{\sigma(j)}$ *for any* $j = 1, 2, \ldots n$ *and* $\mathrm{Ny}(\alpha_{\sigma(j)}) = \sum_{h=1, h\neq j}^{n} Sup(\alpha_{\sigma(j)}, \alpha_{\sigma(h)})$

and $w = (w_1, w_2, \ldots, w_n)^T$ *is the weight vector of* $\alpha_{\sigma(j)} (j = 1, 2, \ldots, n)$, $w \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$.

Theorem 2. *Let* $\alpha j = \langle \mu\alpha j, \eta\alpha j, \nu\alpha j \rangle (j = 1, 2, \ldots, n)$ *be a set of PFN and* $\beta < 0$, *then the aggregated value obtained using PFSSPOWG operator is also a PFN and can be expressed as follows:*

$$PFSSPOWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left\langle \left(\sum_{j=1}^{n} \breve{\chi}_j \mu_j^{\beta}\right)^{\frac{1}{\beta}}, 1-\left(\sum_{j=1}^{n} \breve{\chi}_j (1-\eta_j)^{\beta}\right)^{\frac{1}{\beta}}, 1-\left(\sum_{j=1}^{n} \breve{\chi}_j (1-\nu_j)^{\beta}\right)^{\frac{1}{\beta}} \right\rangle \tag{9}$$

Proof. Proof of this theorem is similar to theorem 1 so we omit here. □

Example 2. *Suppose* $\alpha1 = \langle 0.8, 0.1, 0.05\rangle$, $\alpha2 = \langle 0.79, 0.159, 0.05\rangle$, $\alpha3 = \langle 0.52, 0.23, 0.1\rangle$ *and* $\alpha4 = \langle 0.45, 0.4, 0.01\rangle$ *are four PFNs,* $w = (w1, w2, w3, w4) = \langle 0.2, 0.1, 0.3, 0.4\rangle^T$ *be the weight vector of* αj $(j = 1, 2, 3, 4)$ *then we need to permute the PFs in order to aggregate then using Equation 9. we first calculate the score value of* αj *for* $j = 1, 2, 3, 4$. $Sup(\alpha1, \alpha2) = 1 - \mathrm{D}(\alpha1, \alpha2) = 0.978$, $Sup(\alpha1, \alpha3) = 0.823$, $Sup(\alpha1, \alpha4) = 0.77$, $Sup(\alpha2, \alpha3) = 0.845$, $Sup(\alpha2, \alpha4) = 0.792$ *and* $Sup(\alpha3, \alpha4) = 0.913$. $\mathrm{Ny}(\alpha1) = Sup(\alpha1, \alpha2) + Sup(\alpha3, \alpha1) = 2.571$, $\mathrm{Ny}(\alpha2) = 2.615$, $\mathrm{Ny}(\alpha3) = 2.582$ *and* $\mathrm{Ny}(\alpha4) = 2.475$. $\breve{\chi}1 = 0.201$, $\breve{\chi}2 = 0.102$, $\breve{\chi}3 = 0.304$ *and* $\breve{\chi}4 = 0.393$. *The PFSSPOWG to aggregate the four PFNs and generate a comprehensive value. The steps are shown as follows:* $(\beta = -2)$

$$PFSSPOWG(\alpha_1, \alpha_2, \ldots, \alpha_4) = \left\langle 1-\left(\sum_{j=1}^{4} \breve{\chi}_j (1-\mu_j)^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{j=1}^{4} \breve{\chi}_j \eta_j^{\beta}\right)^{\frac{1}{\beta}}, \left(\sum_{j=1}^{4} \breve{\chi}_j \nu_j^{\beta}\right)^{\frac{1}{\beta}} \right\rangle = \langle 0.317, 0.310, 0.052\rangle.$$

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For a collection of PFN, we can easily prove the proposed PFSSPOWG operator satisfying the idempotency, monotonicity and boundedness properties as follows:

property 4. *(Idempotency) Let* $\alpha_j = \langle \mu_{\alpha_j}, \eta_{\alpha_j}, v_{\alpha_j} \rangle (j = 1, 2, ..., n)$ *be a set of PFN, if* $\alpha_j = \alpha = (\mu, \eta, v)$ *then*

$$PFSSPOWG(\alpha_1, \alpha_2, ..., \alpha_n) = \alpha$$

property 5. *(Monotonicity) Let* $\alpha_j = \langle \mu_{\alpha_j}, \eta_{\alpha_j}, v_{\alpha_j} \rangle$ *and* $\alpha'_j = \langle \mu'_{\alpha_j}, \eta'_{\alpha_j}, v'_{\alpha_j} \rangle$ $(j = 1, 2, ..., n)$ *as two sets of PFN; and* $\mu_j \leq \mu'_j$, $\eta_j \leq \eta'_j$ *and* $v_j \leq v'_j$ *then*

$$PFSSPOWG(\alpha_1, \alpha_2, ..., \alpha_n) \leq PFSSPOWG(\alpha'_1, \alpha'_2, ..., \alpha'_n)$$

property 6. *(Boundedness) Let* $\alpha_j = \langle \mu_{\alpha_j}, \eta_{\alpha_j}, v_{\alpha_j} \rangle (j = 1, 2, ..., n)$ *be a set of PFN, and* $\alpha^+ = (\max_{j=1}^{n} \mu_{ij}, \min_{j=1}^{n} \eta_{ij}, \min_{j=1}^{n} v_{ij})$, $\alpha^- = (\min_{j=1}^{n} \mu_{ij}, \max_{j=1}^{n} \eta_{ij}, \max_{j=1}^{n} v_{ij})$ *then*

$$\alpha^- \leq PFSSPOWG(\alpha_1, \alpha_2, ..., \alpha_n) \leq \alpha^+$$

4 An approach to solving a multi criteria group decision-making problem with the proposed operators

Let $A = \{A_i\}$ $i = 1, 2, \cdots, m$ be a finite set of alternatives and $B = \{B_j\}$ $j = 1, 2, \cdots, n$ be a finite set of criteria which are represented by PFNs. Let $w = (w_1, w_2, \cdots, w_n)^T$ be the weighting vector of the criteria $B = \{B_j\}$ $j = 1, 2, ..., n$ with $w \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Let $\varphi = (\varphi_1, \varphi_2, \cdots, \varphi_r)^T$ be the weighting vector of the decision-makers $C = \{C_s\}$ $s = 1, 2, ..., r$ with $\varphi \in [0, 1]$ and $\sum_{s=1}^{n} \varphi_s = 1$. Now, the procedure for selecting the best alternative using the proposed operators is explained step by step.

Decision matrix $C^s = [\alpha_{ij}]_{m \times n}$

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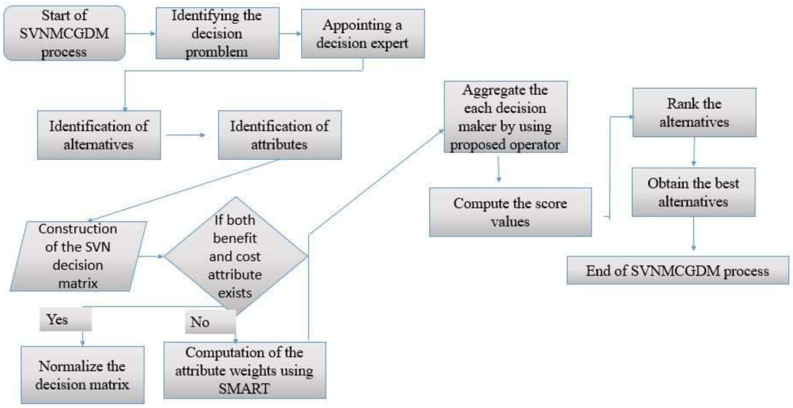

Figure 4: Flowchart for PFMCGDM

$$C^s = \begin{pmatrix} \langle\alpha_{11}\rangle & \langle\alpha_{12}\rangle & \cdots & \langle\alpha_{1n}\rangle \\ \langle\alpha_{21}\rangle & \langle\alpha_{22}\rangle & \cdots & \langle\alpha_{2n}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle\alpha_{m1}\rangle & \langle\alpha_{m2}\rangle & \cdots & \langle\alpha_{mn}\rangle \end{pmatrix}$$

Step 1: Usually, every decision matrix contained two types of data, like benefit $B1$ and cost $B2$ type criterias, if the data is cost type criteria, then normalize the SVN decision matrix using the following Equation 10

$$\varphi_{ij} = \begin{cases} \alpha_{ij}\,; & j \in B1 \\ \alpha^c_{ij}\,; & j \in B2 \end{cases} \tag{10}$$

Where α^c_{ij} is the complement α_{ij}.

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Step 2: Aggregate all s decision matrixs C^s, $s = 1, 2, \cdots, r$ as provided by r experts into a collective decision matrix by employing proposed operators.

Step 3: Determine the weight wj for each criteria Cj . In this step, the SMART is used to establish the criteria weights, relying on the subjective judgment of decision experts. The decision maker ranks the criteria according to their importance, from least to most significant. The criteria deemed least important is given 10 points, while the most important criteria is assigned 100 points. The remaining criteria receive points in ascending order based on their relative importance. Finally, the weight of each criteria is calculated by normalizing the total points so that they sum to one.

Step 4: Aggregate PFNs by using proposed operators.

Step 5: Calculate the score value for each $\{Ai\}$. If $S(Ai) = S(Aj)$ then calculate the accuracy value.

Step 6: The best alternative is selected by ranking the alternatives based on their score and accuracy values of Ai. The proposed framework's workflow is illustrated in Figure 4.

5 Numerical example

The proposed operators are examined through a numerical example from the decision-making field, as outlined below:

This section illustrates the selection of a road construction companies to show the applicability of MAGDM based on the proposed operators. Selecting the right road construction company is a crucial step for governments and enterprises to ensure infrastructure projects are completed efficiently and to high standards. Suppose a government agency is responsible for developing new roads in an urban area. These projects often require careful consideration of multiple factors, including Cost $B1$, Quality $B2$, Time $B3$, Flexibility $B4$, and Environmental Impact $B5$. To evaluate the five companies, $A2$, $A2$, $A3$, $A4$, and $A5$. By applying the developed algorithm for the best road construction companies, the group decision makers process ensures that all necessary steps are followed to choose the most companies. The weights assigned by the decision makers are specified as $\varphi = (0.25, 0.45, 0.3)$. Gather the decision makers' opinions under PF environment are listed in Table 2, 3 and 4 shows the decision matrices.

Table 2: Decision matrix derived from the decision maker C^1

	B_1	B_2	B_3	B_4	B_5
A_1	⟨0.10, 0.20, 0.40⟩	⟨0.60, 0.05, 0.10⟩	⟨0.50, 0.10, 0.33⟩	⟨0.54, 0.23, 0.14⟩	⟨0.60, 0.20, 0.10⟩
A_2	⟨0.10, 0.20, 0.40⟩	⟨0.50, 0.10, 0.10⟩	⟨0.80, 0.04, 0.10⟩	⟨0.55, 0.20, 0.15⟩	⟨0.70, 0.10, 0.05⟩
A_3	⟨040, 0.10, 0.10⟩	⟨0.10, 0.20, 0.40⟩	⟨0.70, 0.10, 0.02⟩	⟨0.50, 0.10, 0.30⟩	⟨0.30, 0.40, 0.20⟩
A_4	⟨0.40, 0.20, 0.30⟩	⟨0.50, 0.10, 0.15⟩	⟨0.10, 0.20, 0.40⟩	⟨0.10, 0.20, 0.40⟩	⟨0.50, 0.20, 0.05⟩
A_5	⟨0.10, 0.20, 0.40⟩	⟨0.20, 0.04, 0.06⟩	⟨0.60, 0.04, 0.05⟩	⟨0.80, 0.03, 0.06⟩	⟨0.25, 0.04, 0.50⟩

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Table 3: Decision matrix derived from the decision maker C^2

	B_1	B_2	B_3	B_4	B_5
A_1	⟨0.10, 0.20, 0.40⟩	⟨0.20, 0.04, 0.06⟩	⟨0.60, 0.04, 0.05⟩	⟨0.80, 0.03, 0.06⟩	⟨0.25, 0.04, 0.50⟩
A_2	⟨0.40, 0.20, 0.30⟩	⟨0.50, 0.10, 0.15⟩	⟨0.10, 0.20, 0.40⟩	⟨0.10, 0.20, 0.40⟩	⟨0.50, 0.20, 0.05⟩
A_3	⟨040, 0.10, 0.10⟩	⟨0.10, 0.20, 0.40⟩	⟨0.70, 0.10, 0.02⟩	⟨0.50, 0.10, 0.30⟩	⟨0.30, 0.40, 0.20⟩
A_4	⟨0.10, 0.20, 0.40⟩	⟨0.50, 0.10, 0.10⟩	⟨0.80, 0.04, 0.10⟩	⟨0.55, 0.20, 0.15⟩	⟨0.70, 0.10, 0.05⟩
A_5	⟨0.10, 0.20, 0.40⟩	⟨0.60, 0.05, 0.10⟩	⟨0.50, 0.10, 0.33⟩	⟨0.54, 0.23, 0.14⟩	⟨0.60, 0.20, 0.10⟩

Table 4: Decision matrix derived from the decision maker C^3

	B_1	B_2	B_3	B_4	B_5
A_1	⟨0.40, 0.20, 0.30⟩	⟨0.50, 0.10, 0.15⟩	⟨0.10, 0.20, 0.40⟩	⟨0.10, 0.20, 0.40⟩	⟨0.50, 0.20, 0.05⟩
A_2	⟨0.10, 0.20, 0.40⟩	⟨0.20, 0.04, 0.06⟩	⟨0.60, 0.04, 0.05⟩	⟨0.80, 0.03, 0.06⟩	⟨0.25, 0.04, 0.50⟩
A_3	⟨040, 0.10, 0.10⟩	⟨0.10, 0.20, 0.40⟩	⟨0.70, 0.10, 0.02⟩	⟨0.50, 0.10, 0.30⟩	⟨0.30, 0.40, 0.20⟩
A_4	⟨0.10, 0.20, 0.40⟩	⟨0.50, 0.10, 0.10⟩	⟨0.80, 0.04, 0.10⟩	⟨0.55, 0.20, 0.15⟩	⟨0.70, 0.10, 0.05⟩
A_5	⟨0.10, 0.20, 0.40⟩	⟨0.60, 0.05, 0.10⟩	⟨0.50, 0.10, 0.33⟩	⟨0.54, 0.23, 0.14⟩	⟨0.60, 0.20, 0.10⟩

5.1 Procedure steps for group decision-making

Step 1: Given that the criteria are all benefit criteria, there is no need to normalise them.

Step 2: To combine decision matrices $C^s, s = 1, 2, 3$ into a collective decision matrix, the PFSSPWG operator is applied, as shown in Table 5, with the decision maker's weighting vector, $\varphi = (0.25, 0.45, 0.43)$.

Table 5: Comprehensive decision matrix C using PFSSPWG operator

	$B1$	$B2$	$B3$	$B4$	$B5$
A_1	⟨0.117, 0.200, 0.375⟩	⟨0.130, 0.200, 0.362⟩	⟨0.400, .100, 0.100⟩	⟨0.114, 0.200, 0.380⟩	⟨0.100, 0.200, 0.400⟩
A_2	⟨0.278, 0.062, 0.099⟩	⟨0.315, 0.084, 0.114⟩	⟨0.100, 0.200, 0.400⟩	⟨0.500, 0.100, 0.113⟩	⟨0.351, 0.048, 0.091⟩
A_3	⟨0.178, 0.110, 0.272⟩	⟨0.152, 0.118, 0.264⟩	⟨0.700, 0.100, 0.020⟩	⟨0.207, 0.084, 0.203⟩	⟨0.519, 0.087, 0.285⟩
A_4	⟨0.181, 0.146, 0.225⟩	⟨0.149, 0.159, 0.278⟩	⟨0.500, 0.100, 0.300⟩	⟨0.196, 0.200, 0.235⟩	⟨0.579, 0.194, 0.123⟩
A_5	⟨0.337, 0.142, 0.351⟩	⟨0.381, 0.137, 0.279⟩	⟨0.300, 0.400, 0.200⟩	⟨0.630, 0.128, 0.050⟩	⟨0.415, 0.171, 0.268⟩

Step 3: This step, we calculate the criteria weights using SMART. Experts assign scores ranging from 10 to 100 to each criteria. The normalized criteria weights are then determined by dividing the points assigned to each criteria by the total sum of points. Table 6 displays the points allocated by each expert and the corresponding normalized criteria weights.

Table 6: Criteria weights determined by the expert susing SMART technique.

Criteria	Points assigned by			Sum	Normalized
	C^1	C^2	C^3	of points	weights w_j
B_1	80	80	80	240	0.255319
B_2	90	90	90	270	0.287234
B_3	70	70	70	210	0.223404
B_4	40	30	40	110	0.117021
B_4	40	40	30	110	0.117021
		Total		**940**	**1**

Step 4: Apply the PFSSPWG operator to aggregate all preference values in Table 7.

Table 7: The overall preference value is calculated using the PFSSPWG operator

PFEWG
A_1 $\langle 0.131, 0.234, 0.234\rangle$
A_2 $\langle 0.184, 0.233, 0.233\rangle$
A_3 $\langle 0.204, 0.228, 0.228\rangle$
A_4 $\langle 0.199, 0.234, 0.235\rangle$
A_5 $\langle 0.360, 0.235, 0.235\rangle$

Step 5: Since $S(A1) = -0.336$, $S(A2) = -0.282$, $S(A3) = -0.252$, $S(A4) = -0.269$ and $S(A5) = -0.110$.

Step 6: The alternatives are ranked according to preference. The ranking of the alternatives is as follows:

$A5 > A3 > A4 > A2 > A1$.

6 Comparative analysis

To illustrate the significance and applicability of the results, we compared the proposed MCGDM approach with existing AOs.

6.1 Comparison with existing approaches

Table 8 below compares the proposed operators with various existing AOs.

Table 8: A comparison of the proposed operators with several existing approaches

Operators	Parameter	Ranking Order
PFWG (Wei(2017))	Not considered	$A_3 > A_5 > A_4 > A_2 > A_1$
PFOWG (Wei(2017))	Not considered	$A_1 > A_2 > A_3 > A_4 > A_5$
The Proposed AOs		
PFSSPWG	Considered (β=-2)	$A_5 > A_3 > A_4 > A_2 > A_1$
PFSSPOWG	Considered (β=-2)	$A_1 > A_2 > A_3 > A_4 > A_5$

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Table 8 compares the proposed operators with existing aggregation operators. The suggested operators yield different ranking outcomes compared to the existing ones. Therefore, the proposed operators can serve as a more effective alternative to the existing operators.

Based on the comparative study above, the proposed approach for tackling decision-making challenges offers several advantages over the existing operators.

As observed in Table 8, the results obtained from various existing approaches are derived in an environment that does not take into account the SS power of the criteria during evaluation. In other words, these approaches assume that decision-makers consider the alternatives being assessed. However, in real-life situations, such conditions are only partially met. Additionally, some existing operators for PFS are specific cases of the proposed operators. Consequently, it can be concluded that the proposed aggregation operators are more suitable for accurately addressing real-life problems compared to the existing ones.

7 Conclusions

This study presents the SMART for addressing MAGDM problems involving PF decision values. Aggregating decision values is a critical initial step in PFMCGDM. We introduce a series of PFSSPG operators, starting with the PFSSPG operator and its desirable properties. We also develop weighted variants, including PFSSPWG and PFSSPOWG operators and examine their fundamental properties. The SMART is employed to determine criteria weights based on decision experts' subjective knowledge. A real-life supplier selection application in the chemical industry demonstrates the practical applicability of the proposed work. A comparative study with existing oper- ators highlights the feasibility and flexibility of our solution methodology. Future research will focus on applying the developed AO to average, linguistic and complex uncertain environments, exploring various PF AOs, and applying them to practical decision scenarios to enhance our understanding and effectiveness in handling diverse fuzzy DM situations.

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